A High-Accuracy Four-Position Tilt Mechanism

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ABSTRACT

Conceptual and dimensional synthesis of a high-accuracy four-position tilt mechanism is discussed in detail. The mechanism, which consists of a customized five-bar linkage, is capable of accurately rotating an optical bench, which supports a space instrument, to four required and discrete angular postures. The mechanism is driven by two stepper motors. It is shown that, due to the special characteristics of the five-bar linkage, even relatively large stepper motor errors produce very minor errors in the four desired angular postures of the optical bench. The dimensional synthesis of the mechanism involves solving a system of four non-linear equations in four unknowns. Such a system of equations can be solved numerically. Additionally, it is shown that if there are only three required optical bench angular postures, the dimensional synthesis of the mechanism can be reduced to a closed-form quadratic equation in one unknown. Examples of both the general numerical solution and the special closed-form solution are discussed in detail.

1. INTRODUCTION

Several discrete positioning systems have been studied in the literature. For example, Sclater and Chironis have discussed a transport linkage with intermittent movements [1]. Brooks has introduced a high-accuracy discrete positioning device [2].

In this paper, conceptual design and dimensional synthesis of a tilt mechanism, which is able to tilt an optical bench to four required angular postures, is presented. The optical bench is to be used for support a space instrument. The tilt mechanism consists of a high-accuracy two-degree-of-freedom five-bar linkage. Two stepper motors will act as drivers for the linkage. The details of the linkage and a procedure for determination of its dimensions are discussed in detail.

2. DESCRIPTION OF THE MECHANISM

Figures 1 through 4 depict the four required tilt configurations of a space optical bench. The tilt mechanism for the optical bench consists of crank O (driven by a fixed stepper motor at point O), crank Q (driven by a moving stepper motor at point Q), a coupler link AB (hinged to cranks O and Q at points B and A, respectively), and the output link PQ (the optical bench). Point P represents the tilt pivot point. Note that in all four configurations, the hinge points A, B, O, and Q are on a straight line. This means that cranks O and Q are either in a “top dead center” or “bottom dead center” location. As a result, even relatively large stepper motor errors produce very minor errors in angular location of the output link PQ. We can accurately position the optical bench at the four required tilt configurations even if the stepper motors are relatively imprecise (without expensive encoders). This improvement in positioning accuracy is due to the fact that when the hinges A, B, O, and Q are linearly aligned, the rate of change of distance between the revolute joints O and Q with angular orientation of the cranks is relatively very small. This rate of change is much larger when the revolute joints A, B, O, and Q are far from linear alignment.

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Three-dimensional views of the four tilt configurations of the optical bench are shown in Figures 5 through 8.

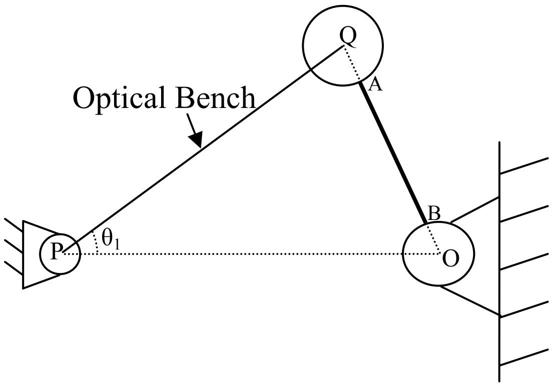

Figure 1 – Depiction of the highest tilt configuration

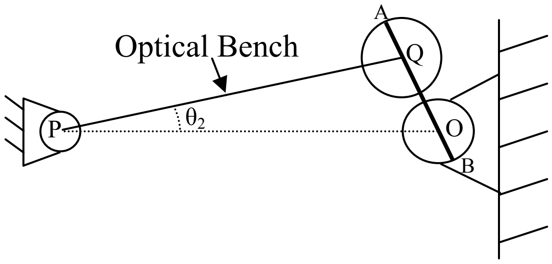

Figure 2 – Depiction of the lowest tilt configuration

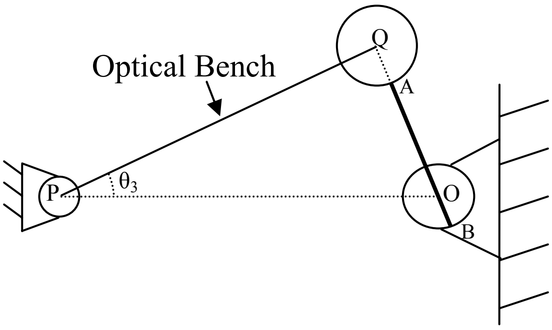

Figure 3 – Depiction of the first intermediate tilt configuration

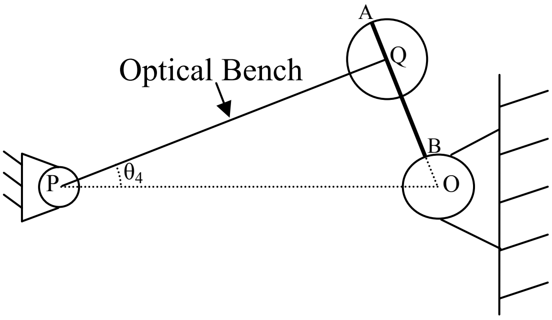

Figure 4 – Depiction of the second intermediate tilt configuration

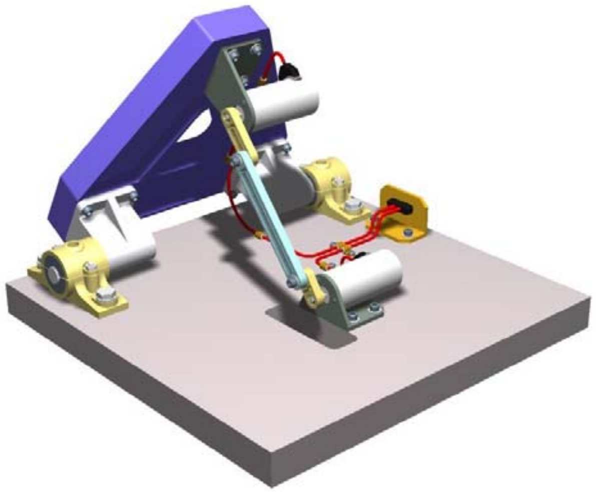

Figure 5 – Three-dimensional view of the highest tilt configuration

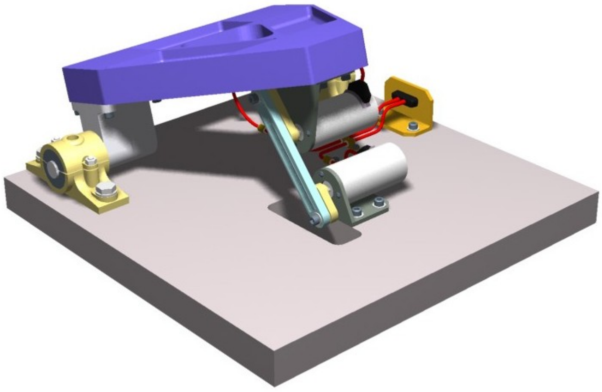

Figure 6 – Three-dimensional view of the lowest tilt configuration

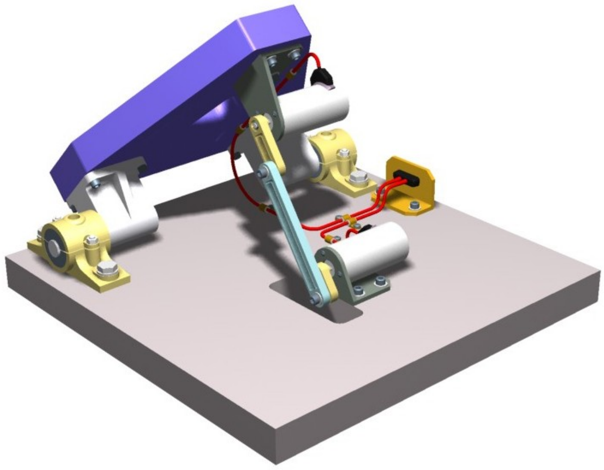

Figure 7 – Three-dimensional view of the first intermediate tilt configuration

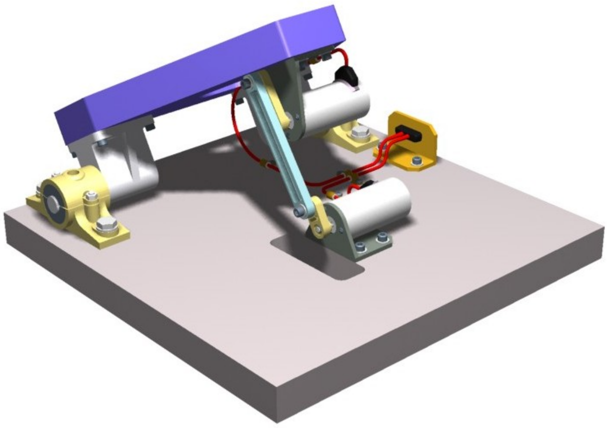

Figure 8 – Three-dimensional view of the second intermediate tilt configuration

3. DIMENSIONAL SYNTHESIS

As shown in Figures 1 through 4, let the OPQ angle in the highest, the lowest, the first intermediate, and the second intermediate tilt configurations be equal to θ_1, θ_2, θ_3, and θ_4, respectively. Also, let

- R_1 = radius of crank O
- R_2 = radius of crank Q
- L = Fixed length OP
- D = Fixed length PQ
- T = Fixed length of the coupler link AB

Using the Law of Cosines, and referring to Figures 1 through 4, we can write the following 4 equations.

$$(T + R_1 + R_2)^2 = D^2 + L^2 - 2DL\cos(\theta_1) \quad (1)$$

$$(T - R_1 - R_2)^2 = D^2 + L^2 - 2DL\cos(\theta_2) \quad (2)$$

$$(T - R_1 + R_2)^2 = D^2 + L^2 - 2DL\cos(\theta_3) \quad (3)$$

$$(T + R_1 - R_2)^2 = D^2 + L^2 - 2DL\cos(\theta_4) \quad (4)$$

The four desired tilt angles (θ_1, θ_2, θ_3, and θ_4) are known. Hence, there are 4 equations and 5 unknowns (T, R_1, R_2, D, and L). We have to assign values to one of the variables and solve for the other three. Since the length of the optical bench (dimension D) is a function of the size of the instrument that it is supporting, a good design decision is to assign a value to D and solve for T, R_1, R_2, and L. The MathCAD software package [3] was used to numerically obtain the solutions to this system of four equations in four unknowns. Since Equations (1) through (4) are quadratic, there are two possible set of solutions.

Note that if desirable, it is also possible to assign a value L and let D be one of the four unknown variables.

3.1. Example of the Numerical Solution

Let the required tilt angles θ_1, θ_2, θ_3, and θ_4 be equal to 60, 15, 45, and 30 degrees, respectively. If D=1 meter, then we obtain the following values for the remaining variables.

- T = 0.734 meters
- R_1 = 0.126 meters
- R_2 = 0.256 meters
- L = 1.204 meters

or

- T = 0.609 meters
- R_1 = 0.105 meters
- R_2 = 0.213 meters
- L = 0.831 meters

Note that only the ratios of the lengths D, T, R_1, R_2, and L are important; therefore, D is set equal to 1 meter.

3.2. Special Case with Closed-Form Solution

If there are only three required optical bench tilt angles, we can use the following methodology to come up with a closed-form solution for the dimensional synthesis of the tilt mechanism.

Figure 9 shows the lowest tilt configuration for the special case. Referring to Figure 9, we can observe that

$$T = 2(R_1 + R_2) \qquad (5)$$

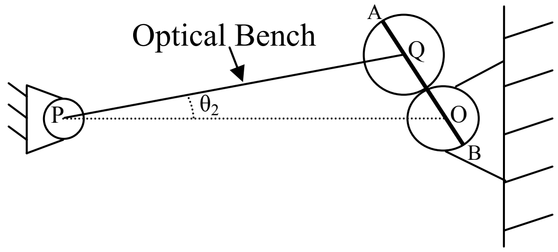

Figure 9 – Depiction of the lowest tilt configuration for the special case

Substituting Equation (5) into Equations (1) through (3) and simplifying, we obtain

$$9\left(R_1 + R_2\right)^2 = D^2 + L^2 - 2DL\cos\left(\theta_1\right) \qquad (6)$$

$$\left(R_1 + R_2\right)^2 = D^2 + L^2 - 2DL\cos\left(\theta_2\right) \qquad (7)$$

$$\left(R_1 + 3R_2\right)^2 = D^2 + L^2 - 2DL\cos(\theta_3) \qquad (8)$$

Subtracting Equation (6) from 9 times Equation (7), and simplifying, we get

$$4L^2 + \left[\cos\left(\theta_1\right) - 9\cos\left(\theta_2\right)\right]DL + 4D^2 = 0 \qquad (9)$$

As discussed earlier, we can assign a value to D and solve for L. Note that L can only be positive. Next, we can substitute the L value in Equations (7) and (8) to obtain positive values for $(R_1{+}R_2)$ and $(3R_1{+}R_2)$. We can then use these $(R_1{+}R_2)$ and $(R_1{+}3R_2)$ values to solve for R_1 and R_2 (2 equations in 2 unknowns). Finally, having found R_1 and R_2, we can use Equation (8) to solve for T.

As mentioned before, it is also possible to assign a value L and let D, R_1, R_2, and T be the four unknown variables.

3.2.1. Example of the closed-form solution (special case)

Let the three required tilt angles θ_1, θ_2, and θ_3 be equal to 70, 20, and 45 degrees, respectively. If D=1 meter, using the above procedure, we obtain the following values for the remaining variables.

- L = 1.185 meters
- R_1 = 0.204 meters
- R_2 = 0.216 meters
- T = 0.421 meters

or

- L = 0.844 meters
- R_1 = 0.173 meters
- R_2 = 0.183 meters
- T = 0.355 meters

As mentioned earlier, only the ratios of the lengths D, L, R_1, R_2, and T are important; therefore, D is set equal to 1 meter.

4. REFERENCES

[1] Neil Sclater and Nicholas P. Chironis, *Mechanisms and Mechanical Devices Sourcebook*, Third Edition, pp. 65-67, McGraw Hill, 2001.

[2] John J. Brooks, *High-Accuracy Discrete Positioning Device*, United States Patent Number 5,349,879, 1994.

[3] MathCAD User's Guide, Version 12, Parametric Technology Corporation, Needham, MA, USA.