Force and Torque Margins for Complex Mechanical Systems

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Abstract

Force and torque margins are commonly used within the aerospace community to determine if the actuator for a mechanism has sufficient force or torque to ensure successful operation. Typical mechanism functions include deploying an antenna, releasing a launch restraint, rotating a solar array, controlling an antenna pointing mechanism, operating a valve, or releasing an interface connector. The definition of force margin relates to the ratio between the driving forces and the resisting loads. For example, if the driving force is twice the resisting load, the force margin is 100 percent. The calculation of force and torque margins is relatively straightforward for a simple mechanism such as a spring-loaded hinge. However, for complex mechanisms employing gear trains, linkages, and jackscrews, the basic arithmetical process becomes more complex.

The method described herein references drive forces, and resisting forces and moments, as equivalent forces and moments at a selected point in the mechanism. This is done by multiplying each force or moment by the ratio of its displacement to the displacement at that selected coordinate point. These equivalent forces from the various points of the mechanism are then summed at the selected common point, keeping driving forces separate from resisting forces, for use in the basic force and torque margin formulas. It is shown that force and torque margins can be calculated as energy margins, power margins, or virtual work margins, and that these margins have a simple relationship to mechanical efficiency.

Introduction

Force and torque margins are the functional counterparts of structural safety factors and stress margins. Functional margins as such are relatively new, having been instituted three decades ago because of failures of launch vehicle and spacecraft mechanisms to operate as intended. The definition of force margin relates to the ratio between the driving forces and the resisting loads. If the driving force is greater than the resisting load, the mechanism will start to move with an initial acceleration that is a function of the magnitude of the margin. For example, if the driving force is twice the resisting load, the force margin is 100 percent.

Typical spacecraft and launch vehicle mechanism functions include deploying an antenna, releasing a launch restraint, rotating a solar array, controlling an antenna pointing mechanism, operating a valve, or releasing an interface connector. The drive force or torque is typically provided by a spring-driven actuator, a pneumatic or hydraulic piston, an electromechanical device such as a motor or solenoid, or thermal expansion from an expanding wax actuator or shape memory alloy.

The resisting forces or torques typically include friction from sliding surfaces, bearings, and gears. The internal friction and the stiffness of wire harnesses routed across moving joints or interfaces are a major source of mechanical resistance in a deployable device. The type of friction of greatest concern is static friction (Coulomb friction), which is usually somewhat greater than the sliding friction that follows the onset of motion. Resisting loads can also be due to gravity or acceleration during the launch phase. Valves may have loads due to fluid pressure, and solenoid actuated devices may have loads due to residual magnetism or a return spring.

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Although referred to as a static margin, the calculation involves aspects of mechanical dynamics as well as statics. Of particular interest is the imminent or initial motion of a machine, and in some cases the following continuous motion. One example is a deployable such as a solar array being released from a launch lock and then moving to its final position; another is a scanning mechanism coming to a stop and then reversing direction.

For static margins, the resisting loads do not include velocity-dependent loads from rate controllers. Coulomb friction from bearings, a hysteresis damper, or between a paddle and housing in a viscous damper, would be included, however. The object is to determine if there is sufficient driving force to ensure that the mechanism will start and then not stall at any point in its operational path or cycle.

The calculation of force and torque margins is relatively straightforward for a simple mechanism such as a spring-loaded hinge. However, for complex mechanisms employing gear trains, linkages, or jackscrews, the basic arithmetic process needs special adaptation. For example, the drive actuator can be separated from the driven members by multiple gear meshes having sources of friction drag at various points. The problem becomes more complex if the gear train includes a crank or jackscrew to convert rotary motion to linear motion, which leads to the question as to whether to calculate a force margin or a torque margin.

The method described herein references drive forces and resisting forces, and moments, as equivalent forces and moments at a single selected point in the mechanism. This is done by multiplying each force or moment by the ratio of its displacement to the displacement at that selected coordinate point. The displacement ratios times their respective forces or moments, from the various points of the mechanism, are then summed to collect them as equivalent forces and moments at the selected common point. As an example of the calculation of margins for a complex mechanism, the method will be applied to a motor driving a worm gear, in turn driving a crank and piston through a spur gear. Then the formula will be stated in general terms.

The procedure will be developed using small displacements and mechanical work. The discussion will include the relationship of this approach to the method of virtual displacements and virtual work. It will also be shown that force and torque margins can be calculated as energy margins, power margins, or virtual work margins, and that these margins have a simple relationship to mechanical efficiency.

DEFINITION OF FORCE AND TORQUE MARGIN

Force and Torque Margins were first defined in a USAF "Specification for Moving Mechanical Assemblies for Space Vehicles…" (MMA Specification, Ref. 1, 1975), and twice revised (Ref. 2, 1978 and Ref. 3, 1988). The basic definition is used by NASA in a similar form (GEVS, Ref. 4), and in recent years has been generally adopted by the aerospace industry and incorporated into company design specifications. Static force margins required for mechanisms range from 100 percent (Refs. 1 to 3) to 200 percent (Ref. 4). These are equivalent to safety factors of 2 to 1 and 3 to 1 respectively. In some cases, weight or power restrictions or a manufacturer's past practice has led to the use of margins less than 100 percent.

From Reference 1, static force margin and static torque margin expressed as a percentage, are defined as follows:

$$\text{Static Force Margin} = \left[\frac{\text{Drive Force}}{\text{Resisting Force}} - 1\right] \cdot 100 \tag{1}$$

$$\text{Static Torque Margin} = \left[\frac{\text{Drive Torque}}{\text{Resisting Torque}} - 1\right] \cdot 100 \tag{2}$$

In Reference 3 (1988), a change was made to the formula, in that the force required for acceleration is subtracted from the drive force. This will be discussed later. To minimize repetition, in the following discussions the use of the term force margin will include its counterpart, torque margin, and vice versa.

Extension of Force Margins to Complex Systems

The static force or torque margin as stated in Equations 1 and 2 applies explicitly to the case of a single drive force or torque and a single resisting force or torque at the same translating or rotating point of the mechanism. The equation can also be directly applied when there are multiple driving and resisting forces and torques applied at different points of the same element. The calculation then requires direct summation of all the forces and torques on the common element, the driving forces going in the numerator and the resisting loads in the denominator. This often occurs when driving forces and resisting loads are collinear on the same sliding element, and all of the drive and load points move in the same direction at the same rate.

$$\text{Static Force Margin} = \left[\frac{\sum \text{Drive Forces}}{\sum \text{Resisting Forces}} - 1\right] \cdot 100 \qquad (3)$$

The related situation for rotation occurs when all of the applied forces and resisting loads are on a common shaft. Any linear forces are multiplied by their individual radii relative to the axis of rotation, and thus are easily reduced to torques about the shaft. In this case, the formula for static torque margin would have the same form as Equation 3 above.

If, on the other hand, the mechanism involves multiple gear meshes, or combined rotation and translation from a linkage or jackscrew, the interpretation of the standard force margin formula is not as simple. For a complex mechanism the multiple drive forces and resisting loads at different points of the mechanism need to be related to a single point by the use of multipliers, sometimes called influence coefficients. These coefficients are the displacement ratios or velocity ratios between the elements. To be consistent with the concept of a static force margin, where the displacements and velocities are initially zero, it may be more theoretically correct to speak of virtual displacements, as discussed later. However, the mechanism is in a state of imminent motion if the margin is positive, and the concept of small displacements can be used. If the mechanism comprises only elements that have constant displacement ratios, such as gears and belts, this distinction is less significant. If linkages are included, these ratios will change continuously with the changing geometry of the linkage, and the concept of sufficiently small displacements is important. In the following example, displacements are meant to mean very small displacements.

This discussion of more complex mechanisms assumes a single degree of freedom, and that the motions or imminent motions do not involve flexibility of connecting members. This assumption is implicit in the basic definitions of static torque and force margins. The rotation or translation of these members is referred to using the term “coordinates” to define displacement or velocity vectors. Associated with a single degree of freedom, the displacements and velocities are said to be constrained. For example, two spur gears in mesh would have a rotational coordinate for each gear shaft, and these coordinates are constrained to rotate at rates relative to each other by the ratio of the numbers of teeth (gear ratio). In this case the angular displacement or angular velocity ratio between the elements is the reciprocal of the gear ratio. A 10:1 reduction ratio would result from having ten times a many teeth in the driven gear as in the drive pinion, giving a displacement or velocity ratio of 1:10, relative to the pinion. The torque multiplication ratio, excluding friction drag, is equal to the reciprocal of the displacement ratio, i.e. 10:1. This ratio is sometimes called the mechanical advantage.

Taking a jackscrew as another example, one coordinate would be at the rotation of the drive nut, and the other coordinate the translation of the screw. These two coordinates are constrained to have a displacement ratio that is a function of the lead angle and the pitch diameter of the screw threads. Again, the torque-to-force ratio, excluding friction drag, is equal to the reciprocal of the displacement ratio (i.e. radians/in = lb/in-lb).

Example Procedure

In the procedure to be described, a crank and piston will illustrate change of motion from rotation to translation. A crank linkage will be used, rather than a jackscrew, to illustrate variable mechanical advantage depending on the position of the crank. In Figure 1, a worm gear set, driven by an electric motor, drives a spur gear set and crank linkage. The crank drives a piston having fluid pressure as the resisting load.

Figure 1 also shows the coordinates defining rotation and translation. The driving point coordinate for the rotation of the motor shaft and the attached worm pinion is θ_1. The second coordinate is the driven worm gear and intermediate shaft rotation, θ_2. The third coordinate is the rotation of the crankshaft, θ_3. The rotational coordinates for the linkage bearings are Φ_4 and θ_5. The piston translation is Δ_6. These displacements are referenced to ground, except for Φ_4, which is the rotation of the lower (crankshaft) link relative to the upper (piston) link.

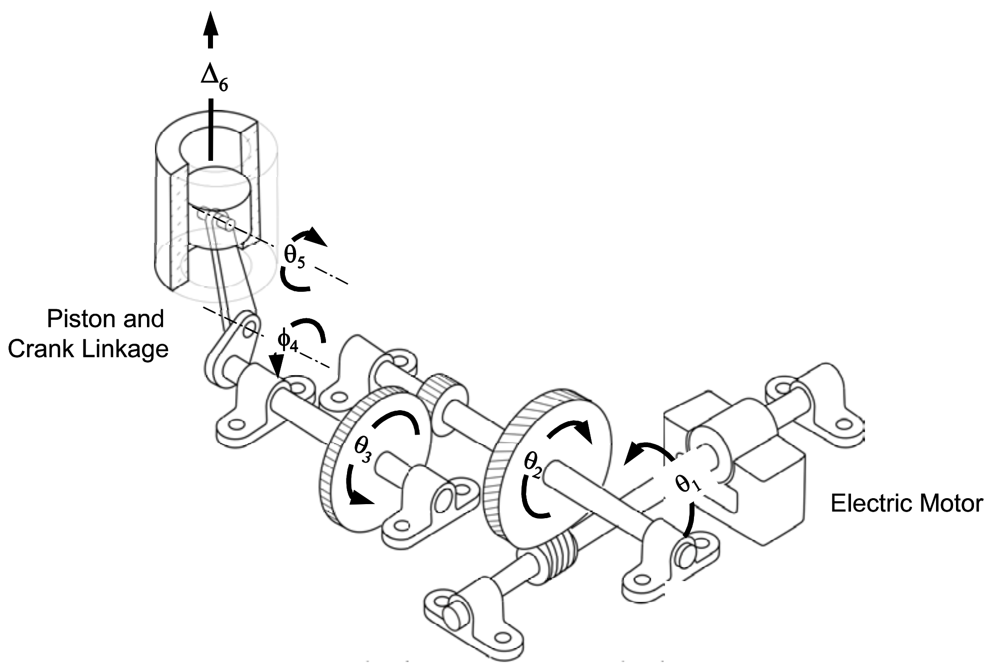

Figure 1
Displacement Coordinates

The driving torque and resisting torques and forces are shown in Figures 2 through 4 as external forces on the shafts, gears, linkage, and piston. The exception is the resisting torque, T_{4B}, at the crank bearing, Coordinate 4, which represents a relative displacement, Φ_4. This resisting torque, T_{4B}, is an interface torque between the upper and lower crank links.

These figures represent portions of Figure 1, and are not free body diagrams showing reaction forces in opposition between the diagrams. It is that type of more complex representation and analysis procedure that this method being presented endeavors to avoid.

<u>Reference Point, Coordinate 1, Motor Drive Shaft Loads (Fig.2)</u>

The reference point for the calculation of equivalent loads will be chosen arbitrarily as the motor pinion drive shaft, θ_1. The driving torque applied to this coordinate (point) is provided by the electromagnetic torque, T_{1M}, on the motor armature.

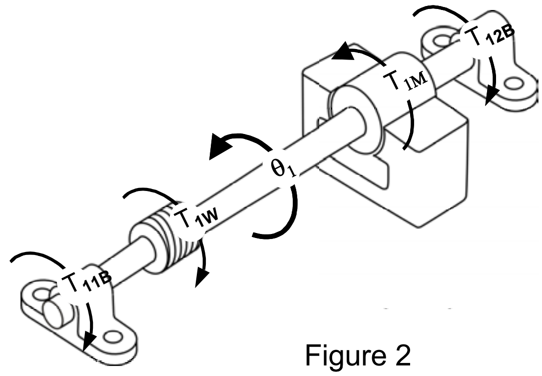

Figure 2
Drive Torques and Resisting
Torques on Drive Shaft

The resisting friction torques, T_{11B} and T_{12B}, from each of the two drive shaft bearings, act in the direction opposite to the drive torque. The drag forces from friction at the worm and gear tooth contact point will be represented as a resisting torque, $T_{1W,}$ acting on the driveshaft. Thus far, we have the friction torque load on the driveshaft as the sum of three components.

$$T_{1R} = T_{11B} + T_{12B} + T_{1W} \tag{4}$$

(Bearing drag torque is partially a function of the reaction force at the bearing, which is in turn, a function of the drive torque. Thus, the values of bearing drag torque would need to be consistent with the motor drive torque, and would be calculated as the solution to simultaneous equations. This also applies to the worm gear friction torque, which likewise depends on the drive torque and can be represented by the efficiency of the gear set times the torque on the worm pinion. This friction torque is in effect applied to the worm but not to the gear).

Coordinate 2, Intermediate Shaft Loads (Fig. 3)
Coordinate 2 is represented by θ_2 at the intermediate gear shaft. As shown in Figure 3, this coordinate is also common to the spur gear pinion.

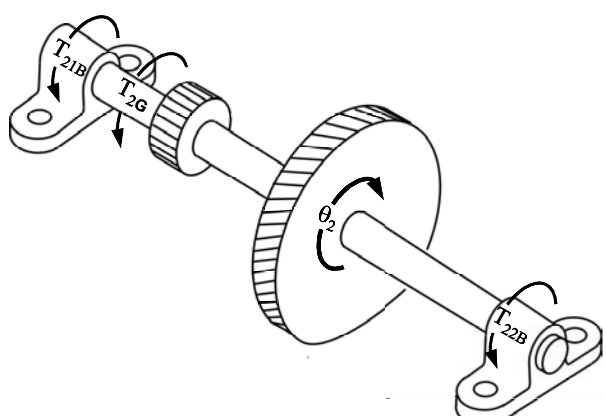

Figure 3
Resisting Torques on Intermediate Shaft

According to this methodology, the torques on this second shaft will be referenced to the selected drive point coordinate by the displacement ratio, θ_2/θ_1. There are two friction torques from the shaft bearings, T_{21B} and T_{22B}. The drag torque from sliding friction at the spur gear teeth will be included with the drag torque on this shaft as T_{2G}. (The friction torque on a spur gear varies with the relative positions of the contacting gear teeth, and is zero when contact is at the pitch line. However, since this friction is small compared to bearing friction, it can be represented as a constant efficiency times the torque on the pinion. The friction torques at the two bearings are likewise a function of the torque on the intermediate shaft). The three friction torques on this shaft are summed to give the resisting torque at Coordinate θ_2.

$$T_{2R} = T_{21B} + T_{22B} + T_{2G} \tag{5}$$

To reference this net drag torque summation at Coordinate 2 to the drive point at Coordinate 1, it is multiplied by the displacement ratio θ_2/θ_1.

$$T_{2/1R} = T_{2R} (\theta_2/\theta_1) \tag{6}$$

The quantity $T_{2/1R}$ constitutes an equivalent or virtual load at the drive point. The term “virtual” here is based on the dictionary definition, "being in effect, but not in fact". (This general definition of “virtual” has a more restricted meaning later when we apply it to virtual work and virtual displacements). In other words, the effect on the acceleration of the mechanism is the same as if the original drag loads at the intermediate shaft were replaced by their equivalent load $T_{2/1R}$ at the drive shaft. The key to the definition of the equivalent loads is that they must do the same amount of positive or negative mechanical work as the original loads. By this process, the loads from the intermediate shaft are referenced to the drive shaft, so that the definition of torque margin for a single rotating coordinate can be used.

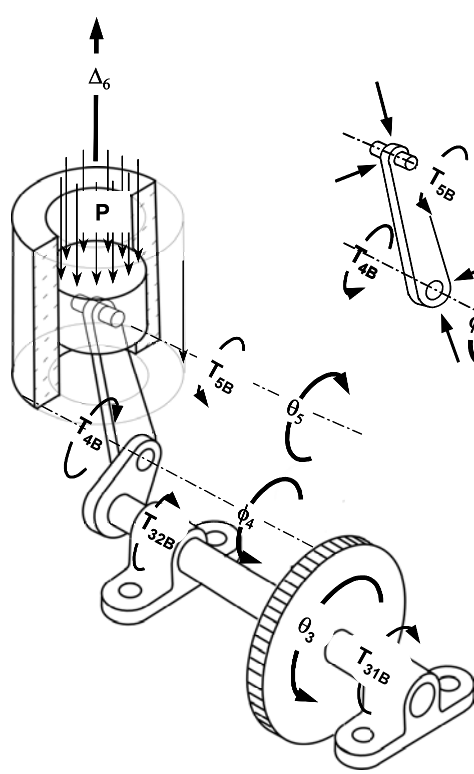

Figure 4
Resisting Forces and Torques
on Crank Shaft and Piston

Coordinate 3, Crankshaft Loads (Fig. 4)
The intermediate shaft drives a crankshaft with angular Coordinate θ_3, having two bearing drag torques T_{31B} and T_{32B}. These two drag torques could be referenced directly to the drive shaft, skipping the intermediate shaft (θ_2), by their displacement ratio to the drive shaft (θ_3/θ_1). However, this task is more systematically organized by first collecting the loads from the crank and piston, and then referencing their sum from the crankshaft (θ_3) to the driveshaft (θ_1).

Coordinates 4, 5, and 6, Linkage Bearing Loads and Piston Loads (Fig. 4)
Although the friction torque from the intermediate crank bearing T_{4B} also acts directly on the crankshaft, this bearing's displacement, Φ_4, is relative to its two mating crank linkages. This relative displacement, Φ_4, is greater that the displacement θ_3 of the lower link relative to the crankshaft, in the crank position shown.

$$\Phi_4 = \theta_3 + \theta_5 \quad (7)$$

The friction torque of this crank bearing acts on both the upper crank and the lower crank. Thus, the combined effect of this friction torque on the crankshaft is greater than the reaction torque of only this bearing on the shaft. The reason can be seen from the free body diagram of the upper link in Figure 4. The reaction of this bearing friction torque on the upper crank link results in a moment on this upper link with an associated couple and a reaction force against the lower link. Thus the energy dissipated by this friction torque is increased by the relative rotation of the upper crank linkage. This is one example of the simplification inherent in this energy method. It avoids the need for separating the crank linkage, crankshaft and piston bearing into three separate free body diagrams.

The effect of the friction torque, T_{4B}, of this bearing on the crankshaft is determined by multiplying it by the angular displacement ratio (Φ_4/θ_3).

$$T_{4B/3} = T_{4B} (\Phi_4/\theta_3) \quad (8)$$

The drag torque T_{5B} on the piston bearing is referenced to the crankshaft in the same way, using the angular displacement ratio of the upper crank linkage to the crank, θ_5/θ_3.

$$T_{5B/3} = T_{5B} (\theta_5/\theta_3) \quad (9)$$

Since the direction and magnitude of Φ_4 and θ_5 are variable with position, their displacement ratios are also variable. Note that for some positions of the crank, Φ_4 is less in magnitude than θ_3.

The fluid pressure force on the piston is equal to the pressure, P, times the piston area, A. The equivalent load could be directly referenced to the motor driveshaft, Coordinate 1, by the ratio of the linear displacement of the piston, Δ_6, divided by the angular displacement of the motor shaft, θ_1. However, to be consistent with the previous treatment of the two crank bearings, this force will first be referenced to the crankshaft by the ratio between Δ_6 and θ_3, and then referenced to the θ_1 coordinate by the ratio between θ_3 and θ_1. Likewise, the drag load from the friction, F_{6P}, between the piston and its cylinder is referenced to Coordinate θ_3 by this same displacement ratio. The equation for the forces and torques referenced to the crankshaft is as follows.

$$T_{3R} = (T_{31B} + T_{32B}) + T_{4B} (\theta_3 + \theta_5)/\theta_3 + T_{5B} (\theta_5/\theta_3) + (P \cdot A + F_{6P})(\Delta_6/\theta_3) \quad (10)$$

If these were referenced directly to the motor drive shaft, the result would be:

$$T_{3/1R} = (T_{31B} + T_{32B})(\theta_3/\theta_1) + T_{4B} (\theta_3 + \theta_5)/\theta_1 + T_{5B} (\theta_5/\theta_1) + (P \cdot A + F_{6P})(\Delta_6/\theta_1) \quad (11)$$

The ratio Δ_6/θ_3 constantly varies throughout the cyclical motion of the piston, as does the fluid pressure P. Note that the algebraic sign of the friction force, F_P, between the piston and cylinder would also change as the piston changes direction at the top and bottom of its stroke. Likewise, the contribution of these torques and forces to the final torque margin also varies throughout the cycle of motion.

Because of the varying displacement ratios, it is important that the displacements be taken over sufficiently small increments such that any errors introduced are negligible. If exact formulas for the kinematic relationship between the piston motion and the crankshaft are used, the derivatives of the individual displacements to the reference displacement θ_1 can be used as the displacement ratios.

Final Static Torque Margin

The static torque margin equation for this example now comprises the drive torque at the drive pinion divided by the resisting torques on the motor shaft, plus the other resisting loads from the intermediate shaft and crankshaft referenced to Coordinate 1 as equivalent loads.

$$\text{Static Force Margin} = \left[\frac{T_{1M}}{T_{1R} + T_{2/1R} + T_{3/1R}} - 1\right] \cdot 100 \qquad (12)$$

General Force and Torque Margin Formulas

The static margin derived for this example can be stated in general as the sums of drive forces divided by the sums of resisting loads. Each drive force or torque and resisting load is multiplied by its displacement ratio, and each is referenced to one arbitrarily chosen coordinate, shown in the following equation as Coordinate j. The displacement ratios can be called influence coefficients, k_{ij}. Using F to represent either driving force or torque and L to represent either resisting force or torque, the static margin is defined as:

$$\text{Static Force or Torque Margin} = \left[\frac{k_{1j}F_1 + k_{2j}F_2 + \ldots\ldots + k_{mj}F_m}{k_{1j}L_1 + k_{2j}L_2 + \ldots\ldots + k_{nj}L_n} - 1\right] \cdot 100 \qquad (13)$$

where m equals the number of driving points,
n equals the number of resisting points,

The F_i and L_i terms represent the sums of driving forces and torques or resisting loads at the respective coordinates, or already referenced to those coordinates from other coordinates by related influence coefficients. The influence coefficients, k_{1j}, k_{2j}, k_{3j}, etc., are the displacement ratios referenced to element Coordinate j. Using the letter u to represent small displacements, either rotational or translational,

$$k_{ij} = u_i/u_j \qquad (14)$$

where i is the coordinate of interest, and j is the reference coordinate.

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One of the coordinates (i = 1,2,3,....etc) will be at the reference point j. Thus, that particular influence coefficient will be equal to unity.

$$k_{ij} = 1 \text{ for } i = j \qquad (15)$$

As a general note, the reference coordinate j can be a totally fictitious or virtual coordinate (i.e., j=0, having an arbitrarily chosen influence coefficient k_{10} relative to Coordinate 1), as long as the other influence coefficients are consistently referenced to this coordinate (j= 0).

Expressing the summation of Equation 10 in a more general form gives:

$$\text{Static Force or Torque Margin} = \left[\frac{\sum_{i=1,m} k_{ij}F_i}{\sum_{i=1,n} k_{ij}L_i.} - 1\right] \cdot 100 \qquad (16)$$

Force and Torque Required for Acceleration

In the Reference 3 revision of the MMA specification, (1988), the drive torque and force required for acceleration were added to the formulas for static torque and force margins. The requirement for the margin to be 100 percent or greater remained the same as before.

$$\text{Static Torque Margin} = \left[\frac{\text{Drive Torque - Torque Required for Acceleration}}{\text{Resisting Torque}} - 1\right] \cdot 100 \qquad (17)$$

Formulas for Kinetic Force and Torque Margin were also included. The requirement specified was 25 percent.

$$\text{Kinetic Torque Margin} = \left[\frac{\text{Drive Torque - Resisting Torque}}{\text{Torque Required for Acceleration}} - 1\right] \cdot 100 \qquad (18)$$

In these formulas, the drive force and torque required for acceleration are part of the requirement rather than being inherent in the mechanism. Accordingly, the methodology for translating the drive and resisting forces and torques to a common point would not necessarily be needed for these specified acceleration torques. One example of drive torque required for acceleration is the torque required for a stepper motor to overcome detent torque and accelerate the rotor past each individual step. This torque can be calculated by a dynamic analysis or determined from test data. Since this additional resisting torque is considered to be a specification, it can be inserted into the formulas already developed (Eqns.12 through 16). Generally, this detent torque would be relatively small.

Force and Torque Margins as Energy and Power Margins

The general force and torque margin equation can be restated as an energy margin. Taking Equation 13 and restating it terms of the displacement ratios, we have:

$$\text{Static Force or Torque Margin} = \left[\frac{\frac{u_1}{u_j}F_1 + \frac{u_2}{u_j}F_2 + ... + \frac{u_m}{u_j}F_m}{\frac{u_1}{u_j}L_1 + \frac{u_2}{u_j}L_2 + ... + \frac{u_n}{u_j}L_n} - 1\right] \cdot 100 \qquad (19)$$

This expression can be converted to an incremental energy margin (or virtual work margin) by canceling out the reference displacement u_j, giving:

$$\text{Energy Margin} = \left[\frac{u_1F_1 + u_2F_2 + ... + u_mF_m}{u_1L_1 + u_2L_2 + ... + u_nL_n} - 1\right] \cdot 100 \qquad (20)$$

Or, restated:

$$\text{Energy Margin} = \left[\frac{\text{Drive Energy}}{\text{Resisting Energy}} - 1\right] \cdot 100 \qquad (21)$$

Each of the terms in the numerator of Equation 20 represents the incremental work done by each of the driving forces over each of their respective small displacements. The terms in the denominator represent the incremental work or energy dissipated by friction forces and the incremental potential energy gained by items such as the compressed fluid implied in the illustrations. (Sometimes the resisting energy can turn positive: for example, depending on the position of the crank and piston. Likewise, cable stiffness can assist rather than resist motion during a portion of the cycle. In some cases, the decision has to be made as to whether such terms belong in the numerator or denominator).

The formula for kinetic torque or force margin (Eqn. 20) can likewise be stated as a kinetic energy margin.

$$\text{Kinetic Energy Margin} = \left[\frac{\text{Drive Energy - Resisting Energy}}{\text{Required Kinetic Energy}} - 1 \right] \cdot 100 \qquad \textit{(22)}$$

One example of required kinetic energy would be the energy needed to ensure adequate separation velocity during vehicle staging. The drive energy would be the energy stored in the kickoff springs, and the resisting energy would be the energy required to extract the electrical interface connector pins from their sockets, or any other sources of frictional losses. In this case the displacements would be the full displacement required for complete separation.

If each of the terms in Equation 20 is divided by the time increment for these small displacements u_i, the expression represents the time average power margin over the increment.

$$\text{Power Margin} = \left[\frac{\text{Drive Power}}{\text{Resisting Power}} - 1 \right] \cdot 100 \qquad (23)$$

The fact that Equation 20 can be altered to give Equation 19 by dividing numerator and denominator by any one of the small displacements u_j shows that the selection of reference point is arbitrary. The numerical result is the same regardless whether an energy margin, a power margin, a force margin, or a torque margin is calculated.

When restated as an energy margin, the force or torque margin is a measure of the kinetic energy acquired by the system as it accelerates from rest over a small displacement u_j at some arbitrary coordinate j. The kinetic energy increment over this small displacement is equal to the numerator of Equation 20 minus the denominator. (Equation 22 for kinetic energy margin shows this relationship explicitly for large displacements). Since the torque or force margins are numerically equal to the energy margins, this restatement of the torque or force margin as an energy margin is an argument for the validity of this method. Thus, the “net force” available for accelerating the system from rest using the equivalent force system, referenced to one coordinate location, is the same as for the original force system. The validity of this method is discussed further under the subject of virtual work.

Mechanical Efficiency

The efficiency of a mechanical transmission system is defined as the ratio of the output work divided by the input work. Mechanical efficiency is typically illustrated in textbooks by an example of a relatively inefficient device, such as a jackscrew or worm drive. This usage has similarities to the force and energy formulas as developed here, except that for mechanical efficiency the work output to the driven device is excluded from the resisting energy. Revising Equation 21 to include only the energy dissipated as friction:

$$\text{Energy Margin} = \left[\frac{\text{Drive Energy - Friction Energy}}{\text{Friction Energy}} \right] \cdot 100 \qquad (24)$$

$$\text{Mechanical Efficiency} = \left[\frac{\text{Drive Energy - Friction Energy}}{\text{Drive Energy}} \right] \cdot 100 \qquad (25)$$

Expressing Mechanical Efficiency and Energy Margin as ratios rather than as percentages, a simple relationship between the two can be formulated.

$$\text{Energy Margin} = \frac{1}{\dfrac{1}{\text{Mechanical Efficiency}} - 1} \tag{26}$$

For example, if the ratio of Drive Energy to Friction Energy is 4:1, the Mechanical Efficiency is 0.75 (75 percent) and the Energy Margin is 3.0 (300 percent).

Virtual Work and Virtual Displacements

In textbooks, the concepts of virtual displacements and virtual work are commonly developed for analysis of mechanical systems and structures in static equilibrium. A virtual displacement is defined as a fictitious or infinitesimal displacement – sometimes as a very small displacement. The mechanical system is considered to undergo virtual displacements, and the net virtual work that results from the forces acting through their respective virtual displacements is equal to zero.

The type of mechanical system being considered here typically has just been released from a launch restraint or is being energized by a current pulse to a stepper motor. The system is in a state of imminent motion with some finite initial acceleration, and an initial velocity equal to zero. To apply the method of virtual work, this system can be considered to be in a state of dynamic equilibrium, with the inertial forces due to the initial acceleration being in equilibrium with the applied forces and resisting forces (in accordance with D'Alembert's principle). The sum of the net virtual work done by the virtual displacements acting through the original force system and the virtual kinetic energy resulting from the virtual displacements acting through the respective inertial forces, is equal to zero.

From another viewpoint, during a test to measure force margin, the mechanism is sometimes restrained from motion by a load or torque gage, and the measured force or torque is gradually reduced by slowly withdrawing the gage. The measured force or torque at the point of imminent motion is used to calculate the margin. In this case, there is no acceleration until the point of release. Here, the mechanism and the test apparatus are in static equilibrium.

The technical approach using virtual displacements is essentially the same as for the preceding approach using small displacements. The terms “small displacements” and “energy” can be replaced by “virtual displacements” and “virtual work”. The arguments are similar and the conclusion is the same. The net virtual work done by the inertial forces and the sum of the drive forces and the resisting forces, after being referenced to a common point as equivalent or virtual forces, is zero; and the system remains in a state of equilibrium. Thus, the equivalent or virtual force system is equal in effect to the original force system, and the method described herein for calculating force and torques margins is theoretically valid.

If there is a conceptual advantage in describing this method of equivalent forces in terms of virtual work, it is that virtual displacements, being considered infinitesimal, do not involve any approximation due to small changes in mechanical configuration (if linkages are involved). Likewise, there is no need for a conceptual distinction between static friction and dynamic friction.

The virtual work approach was not employed explicitly in the example, partly because the mechanical device is in a state of imminent motion, and real displacements can be considered appropriate for the analysis. Moreover, the increased level of abstraction should not be necessary to the understanding and acceptance of these concepts.

Derivative Notation

If a functional relationship can be established for the displacements, this relationship can be differentiated and the displacement ratios can be calculated as derivatives of the displacements to the reference displacement. Using the letter “v” to represent displacements that are not necessarily small or starting from zero, Equation 14 defining the influence coefficients can be represented in derivative notation.

$$k_{ij} = \frac{dv_i}{dv_j} \tag{27}$$

This approach would be particularly useful where the displacement ratios are not constant, and are describable by a mathematical function, such as for the crank linkage of the example.

Concluding Comments

The presentation of this energy-based method has proceeded from an illustrative example to more general forms of the formulas for force and torque margins. The object has been to adapt these commonly used formulas to complex mechanical systems having elements interconnected by gears, jackscrews, belts, and linkages. In extending the force and torque margin formulas to include energy and power margins, the relationship between force and torque margins and mechanical efficiency has been shown. It is believed that this method is intuitively recognizable to those in the academic field and others who do mechanical analysis. The method is believed to be theoretically correct and consistent with the method of virtual work. It is hoped that the comments regarding the validity of the method are convincing and that this presentation will be helpful to those faced with the types of analyses discussed.

The use of force margins offers somewhat more versatility than mechanical efficiency, because force margins can include resisting loads where mechanical work is done (i.e., motor driven pumps) or potential energy is gained (i.e., a hydraulic or mechanical jack). It is conceivable that a more general understanding could lead to extended usage of force and torque margin criteria in the mechanical engineering community, especially for other critical applications such as actuators for aircraft control surfaces.

References

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