Primer – Stepper Motor Nomenclature, Definition, Performance and Recommended Test Methods

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Abstract

There has been an unfortunate lack of standardization of the terms and components of stepper motor performance, requirements definition, application of torque margin and implementation of test methods. This paper will address these inconsistencies and discuss in detail the implications of performance parameters, affects of load inertia, control electronics, operational resonances and recommended test methods. Additionally, this paper will recommend parameters for defining and specifying stepper motor actuators. A useful description of terms as well as consolidated equations and recommended requirements is included.

Introduction

Stepper Motor Actuators are desired in space mechanisms because of their precise incremental control, yet they are inherently under-damped and susceptible to inertial mismatch. These issues will be addressed in detail. While linear straight-line approximation of Stepper Motor performance may be simulated with simple relationships and equations, actual performance in a system with inertia, friction, and compliance may result in dramatically different performance compared to simulations. Often, performance requirements in specifications do not fully reflect the actual requirements, including torque margin. More importantly, an inadequately designed test set-up or incomplete testing could erroneously hide a latent performance issue that may not be identified until the actuator is integrated at a higher assembly.

Linear Performance Approximation

There are many factors that contribute to the actual dynamic performance of a stepper motor actuator. Simple linear extrapolation, however, yields conservative results that works for the vast majority of applications. The linear approximation may be obtained by determining key motor parameters, which are included in most motor manufacturers' catalogs. For our example, Table 1 delineates the key parameters and values used in our example.

Table 1. Key Motor Parameters

Parameter	Units	Symbol	Value
Motor Constant (Non-Redundant)	mN-m/√ Watt	K_M	34
Motor Inertia	kg-m^2	J_M	7.06E-07
Step Angle at Motor	Degrees/step	$\Delta\theta_M$	30
Simplex No Load Response Rate Constant	RPM/√Watt	K_{RR}	325
Motor Bearing Friction (-20º C)	mN-m	f_{BM}	1.0
Magnetic Coulomb Torque	mN-m	f_{CM}	5.0

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Figure 1, graphs a linear simulation of a geared Stepper Motor Actuator at room temperature and at +50º C. The figure points out key performance parameters and introduces some nomenclature that will be used throughout the paper.

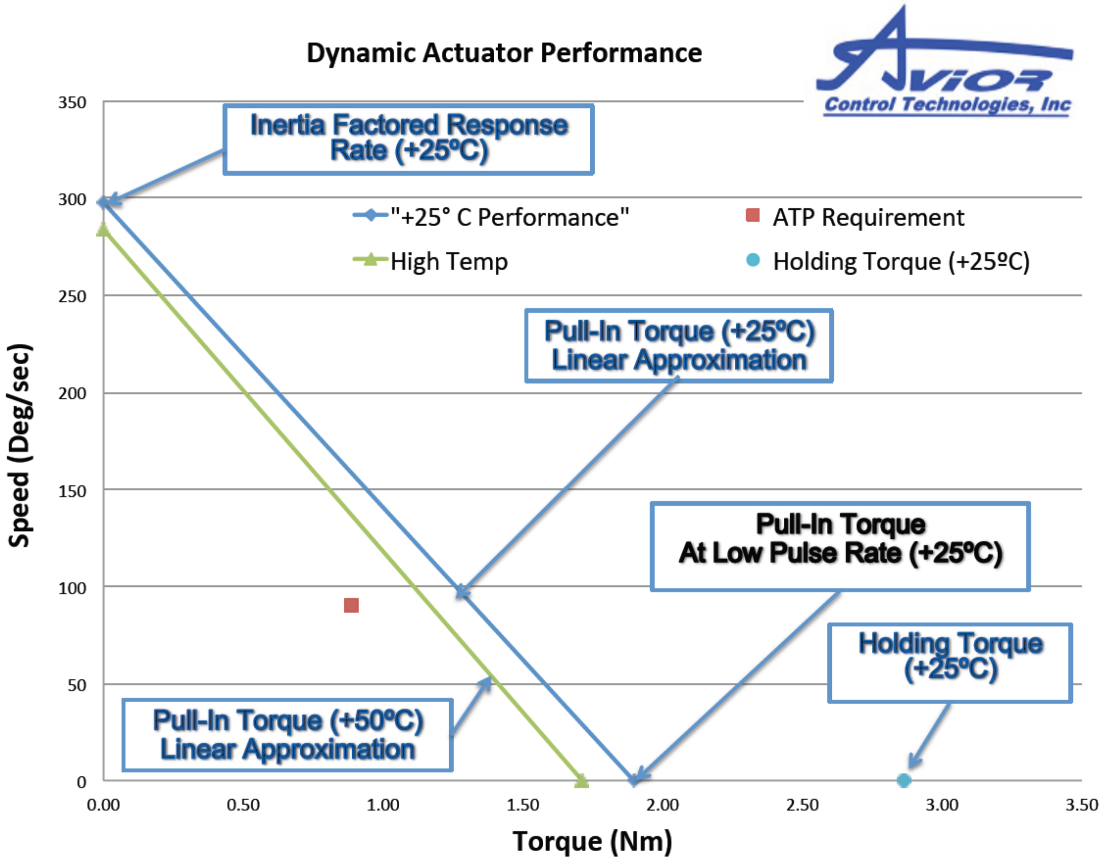

Figure 1. Linear Simulation of Geared Stepper Motor

The designated operating point "ATP Requirement" refers to the required Torque margin requirements, as defined in GSFC-STD-7000 (GEVS) [2]. This will be discussed in detail later in the paper.

Inertia Factor Calculations

Perhaps one of the most important parameters in the utilization of Stepper Motor Actuators is determining the *Inertia Factor* (J_F). This parameter is the sum of the load inertia reflected to the motor (J_{LM}), and the motor inertia (J_M), all divided by the motor inertia:

$$J_F = \frac{J_{LM}+J_M}{J_M} \qquad (1)$$

Where J_{LM} is the load inertia (J_L), divided by the entire gear ratio (N), squared.

$$J_{LM} = \frac{J_L}{N^2} \qquad (2)$$

Many times, the gear ratio is determined by the step resolution required at the system level, but the driving factor may also be reducing the Inertia Factor. There are several schools of thought on what is an acceptable Inertia Factor. Some engineers insist on an Inertia Factor less than or equal to 2.0 (That is $J_{LM} \leq J_M$). While a J_F less than 2.0 is conservative, there are times this may be impractical. A maximum

Inertia Factor less than 5.0 is recommended, but higher reflected inertias may be used with proper testing and analysis.

Response Rate and Torque at Low Pulse Rate Calculations

The Inertia Factored Response Rate (RR_{JF}), or No Load Speed, of a Stepper Motor Actuator is directly effected by the Power Input at Holding (P_H), the Inertia Factor, and Response Rate Constant ($K_{RR'}$). Equation 3 reflects the Inertia Factored Response Rate at the output of the Actuator, in RPM. As a note; the Response Rate Constant for redundantly wound Stepper Motors ($K_{RR'}$) will be higher than a non-redundant winding, because of lower inductive losses (Ldi/dt). The Inertia Factored Response Rate calculation is presented in Equation 3. The factor of 6 converts the units to degrees per second from RPM.

$$RR_{JF} = \frac{K_{RR} \cdot 6.0 \cdot \sqrt{P_{H25}}}{N * \sqrt{J_F}} \quad \text{.......(3)}$$

The Torque at Low Pulse Rate ($T_{PPS\text{-}0}$), presented in Equation 4, is a function of the Holding Torque at 25ºC (T_{H25}) and the sum of the Motor Magnetic Coulomb (f_{CM}) Motor Bearing Friction (f_{BM}) Gearbox Bearing Friction (f_{BG}) Torques, and Gearbox Efficiency (η_G).

$$T_{PPS\_0} = (T_{H25} \cdot 0.707) - N(f_{BM} + f_{CM} + f_{BG}) \quad \text{.......(4)}$$

$$T_{H25} = (N \cdot \eta_G) \cdot K_{M25} \cdot \sqrt{P_{H25}} \quad \text{.......(5)}$$

Equation 4 is a point of discrepancy to be resolved. Professionals in the aerospace industry argue that the Magnetic Coulomb Torque (fc or Detent Torque) is a function of position and typically integrates out over a course of a step. In other words, part of the cycle, the Detent Torque is working against the electro-magnetically generated torque, and part of the cycle, the Detent Torque *works with* the generated torque. Strict interpretation of Torque Margin requirements, these components of torque should be factored as we show in Equation 4, above [2]. *While analyzing **Pull-In Torque,** the detent torque must be subtracted from the available torque to accelerate*. Note, torque margin factors are not applied to torque components such as f_{BM} in this equation because it will be addressed later in the dynamic analysis. The rationale and application of torque margin are discussed in detail in the Comprehensive Torque Margin Analysis section of this paper.

Table 2 delineates the variable values used in our simulation, shown in Figure 1. Data represented in blue herein is a reminder the values are an example.

Table 2. Actuator and System Variables for Sample Simulation

Parameter	Units	Symbol	Value
Load Inertia	kgm^2	J_L	5.7E-04
Holding Power (at +25ºC)	Watts	P_H	28
Gear Ratio	-	N	20:1
Gearbox Efficiency	%	η_G	90
Elevated Temperature	ºC	t_2	50

Performance at Elevated Temperatures

To determine performance of voltage control systems at temperatures other than room temperature (+25ºC) the change in DC Resistance must be determined. The example will analyze a two-phase motor. Refer to supplier catalogs for three phase calculations. For a two-phase bipolar drive, the room temperature DC Resistance (Ω_{25}) per phase may be easily calculated by using Equation 6.

$$\Omega_{25} = \frac{2 \star V^2}{P_{H25}} \quad (6)$$

Where "V" is the supply voltage. In our example, this yields a nominal motor resistance of 48.3 ohms per phase. Use Equation 7 to calculate the DC resistance at temperatures other than 25º C (t_2).

$$\Omega t_2 = \Omega_{25} \; [1 + .004(t_2 - 25)] \quad (7)$$

Knowing the change in resistance at any temperature allows the change in current and power to be calculated. Thus allowing the Torque at Low Pulse Rate and Inertia factored Response Rate to be determined at temperature. Calculations are detailed in Appendix A.

Equation 7 works while analyzing the system is increasing or decreasing temperatures, within limited temperature ranges. Exercise caution when going down in temperature. Colder temperatures could yield higher viscosity in a wet lubrication system and will affect available torque calculations. Avior has modeled multiple wet lubrication options and introduced a functional component of bearing friction (f_{BM}) that accounts for the increased viscosity at colder temperatures. This method is recommended for accurate performance models. Additionally, the relationship of Equation 7 does not work down to cryogenic temperatures where the purity of the copper must be factored.

Actual Dynamic Performance (What Really Happens)

As mentioned, the linear approximation provides a conservative estimate for most applications. Figure 2 shows an empirical test of the Pull-In Torque of the geared stepper motor for the example. It is extremely important to realize that the characteristic oscillations of the dynamic performance will be unique to the test set-up and drive electronics. If high stiffness couplings were replaced with more compliant couplings, the characteristics could dramatically change. The portions of dynamic torque that increase and decrease will be exaggerated with lower compliance, reduced damping, or increased load inertia.

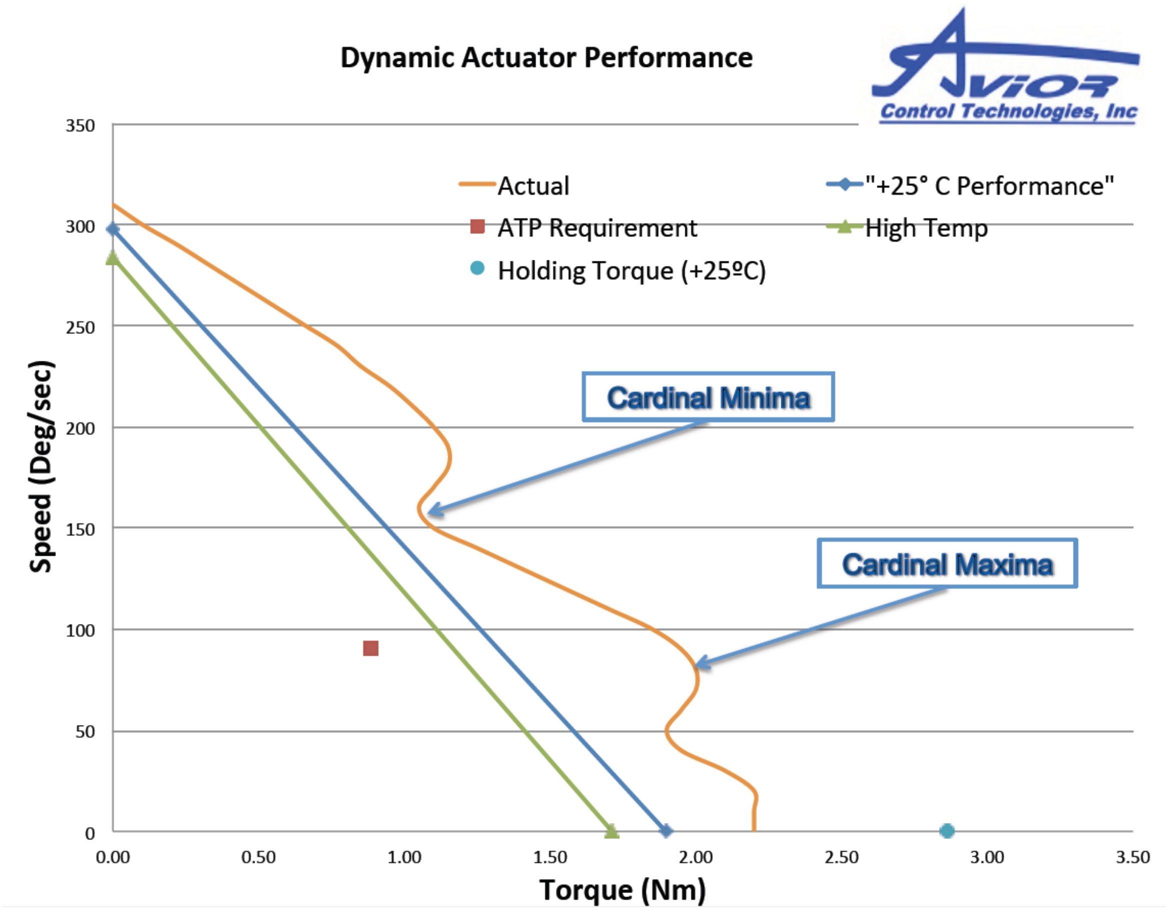

Figure 2. Empirical Dynamic Performance versus Linear Simulation

Why This Happens

The inherent kinematics of a stepper motor mirrors an under-damped step function of a servo system. As the stepper motor moves to each stable step point, there is overshoot of position. Also note that as the motor crosses each step point, the angular velocity, and therefore the kinetic energy, is at a maximum. Even when stepper motors are driven at low pulse rates, the instantaneous angular velocity can be extremely high at these crossover points. The cardinal oscillatory frequencies of these overshoots will result in an increase in torque at some step rates (Cardinal-Maxima), and a reduction of torque at other step rates (Cardinal-Minima). As the Inertia Factor increases, the variation of the torque peaks and valleys will be exaggerated. When the Inertia Factor is greater than 5.0, it is recommended that additional margin be applied from the linear performance assumption before a system prototype has been tested.

There are several important aspects to take into account, considering these phenomena. The characteristic cardinal torque step rates will vary with test set inertia, drive electronics and coupling compliance. This is why it is recommend the drive electronics and test set configuration simulate the actual system parameters as closely as practical. Additionally, performance at multiple step rates should be conducted to verify that dynamic torque performance is not conditionally marginal. In other words, during a test, the system may be at a cardinal torque increase point. If you test multiple step rates around the system operating step rate, you may better characterize the performance. It is important to realize that more torque is not necessarily better. Torque margin is desirable, but increasing torque may actually increase kinematic overshoot and further exaggerate the dynamic cardinal torque variations.

Determining the damping ratio of a Stepper Motor system can be an excellent indicator for susceptibility of a system to extreme cardinal exaggerations. From actual test data, Figures 3 and 4 show the same system with two actuators with different drive methods and damping ratios (ζ). The total inertia, inertia factor, power input and drive systems were identical but the system in Figure 3 was driven bipolar and Figure 4 was controlled with wave drive electronics. This clearly demonstrates the bipolar system provides more damping.

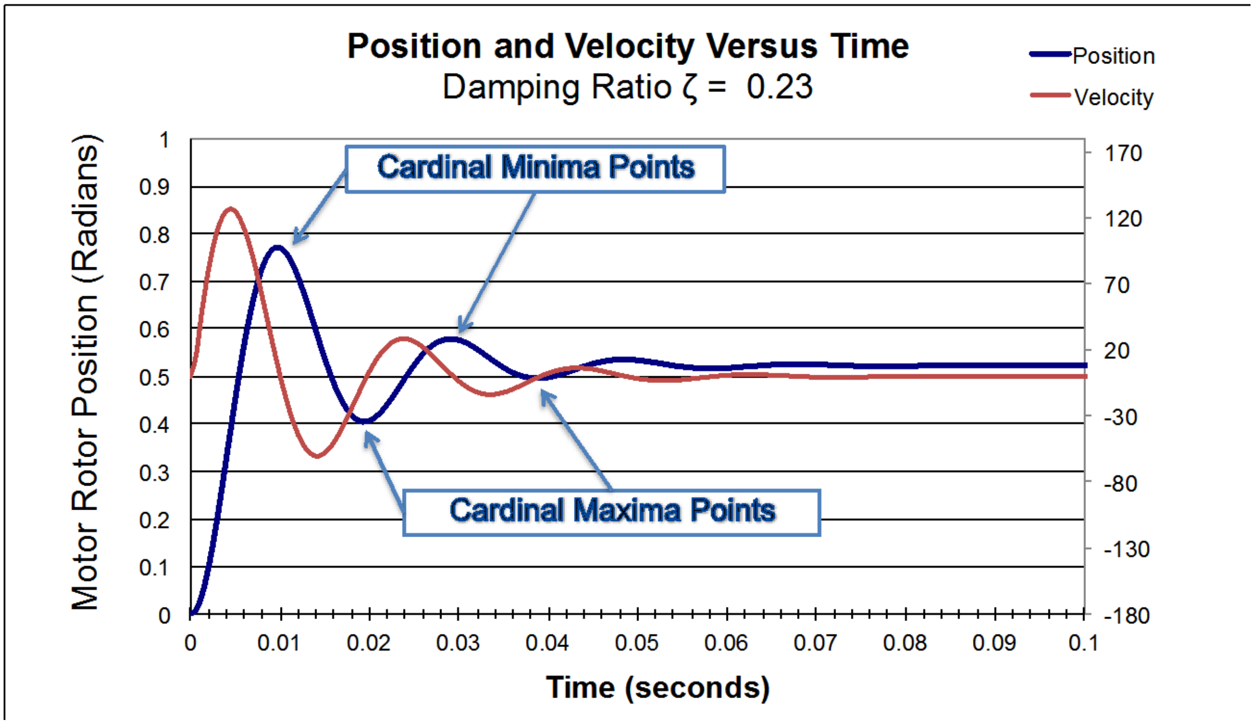

Figure 3. Bipolar Driven Actuator System
Position and Velocity versus Time

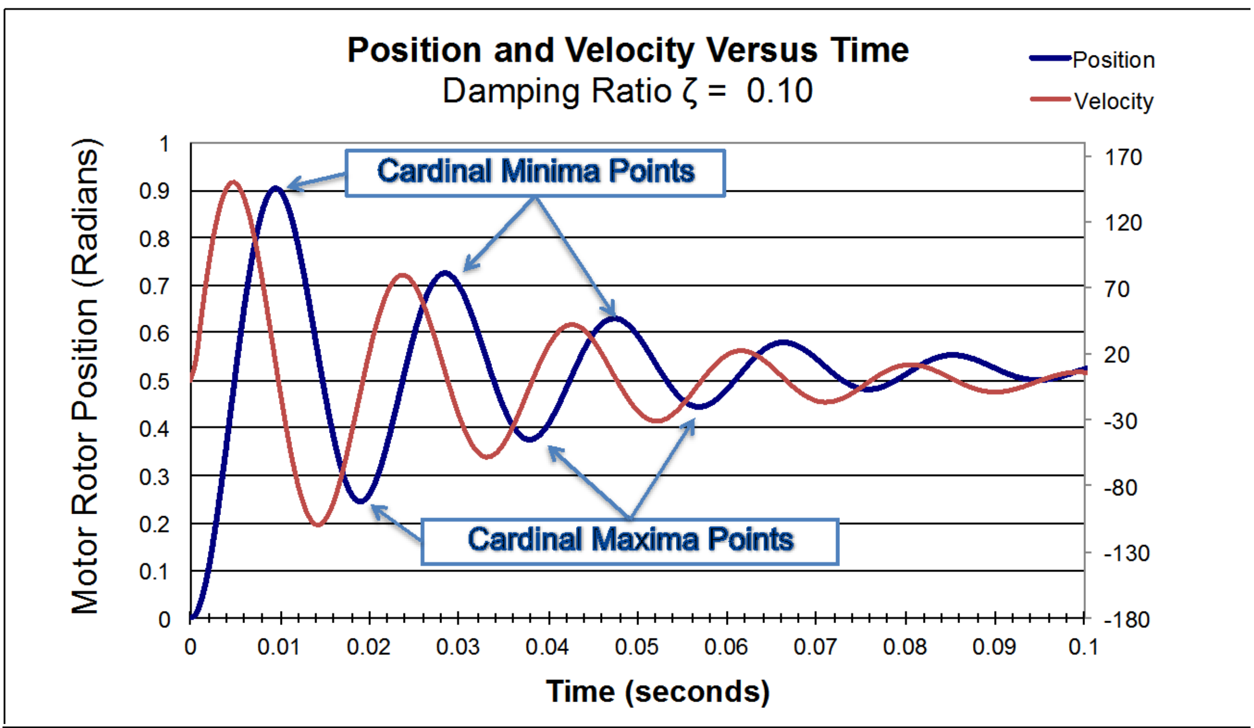

Figure 4. Wave Driven Actuator System

Position and Velocity versus Time

The Cardinal Minima points occur at the peak overshoot points of step because the electromechanically generated torque drops off with the cosine of the position of the overshoot. Obviously, to minimize the impact and number of Minima occurrences, it is desirable to minimize the magnitude and number of overshoots, and the best method to do so is to increase damping.

How to Compensate (How to Increase Damping)

If you find yourself at a conditionally marginal operating rate (or Cardinal-Minima) there are several approaches that may be taken to address the issue. Fundamentally, the main issue with these conditionally marginal operating points is damping. While technically, damping is the loss of torque at speed, an under-damped stepper motor that loses dynamic torque at resonant frequencies can regain torque margin by increasing damping through drive methods or adding electromechanical damping in the actuator. Reference [3] details consequences, options and results of an under-damped system.

Perhaps the two most common causes of under-damped resonant torque losses are drive method or too excess inertia (Inertia Factor). Bipolar drives, whether two phase or three phase, are far superior to unipolar or wave drive methods, in terms of adequately damped systems. As discussed, Inertia Factors greater than 5.0 can tend to have significant resonant torque losses at cardinal-operating frequencies. If bipolar drive is already implemented, then internal electro-mechanical damping methods may be the best solution to remedy under-damped systems. These methods, however, take away from copper volume for motor torque generation, so the effective motor constant, or torque per square root watt, is reduced. Other methods, such as response shaping networks through sensor feedback and processing are effective, but increase the complexity and development of the drive electronics.

Slew Operation (Operating in the Pull-Out Region)

Thus far pull-in torque performance or the torque capacity to pull-in from rest has been discussed. Increased dynamic performance is achievable while operating in the pull-out or slew region. Increased torque capacity or velocity may be achieved by ramping (or slewing) up the step rate of the motor. This will allow the actuator to operate in a performance region that may not be achievable from a dead stop. It is important to not only ramp-up, but also ramp down step rate. It is not advisable to depend on counting steps for positional information if operating in the slew region.

Calculating the linear performance in the slew region is similar to the pull-in analysis described above, but Inertia Factor does not affect the Slew Rate Constant (K_{SR}), as it affects Response Rate Constant (K_{RR}). The pull-out Torque is calculated by a linear approximation from the Slew Rate to the Torque at Low Pulse Rate. Cardinal Oscillatory effects as described above may impact slew operation, just as they effect pull-in operation.

Comprehensive Torque Margin Analysis

Torque Margin has been calculated, defined and interpreted by almost every imaginable method. Many companies have their own methods to define and apply Torque Margin, but as described above and addressed in [3], too much torque can result in under-damped performance. In addition to exaggerating the cardinal minima, too much torque and severely under-damped systems may introduce fatigue and stressing of mechanical components. Additionally, too much torque equates to excess power, unnecessary heat loss and energy consumption. It is certainly prudent to assure torque requirements are satisfied, but this does not mean the design engineer should simply increase torque at a mechanism to ensure a robust system.

Components of Torque

Know your torque contributors and their characteristics. In addition to bearing friction, gear frictions and magnetic coulomb torques, acceleration torques must be accounted for at the step rate of the actuator.

Reference [2] requires different application of torque margin at different stages of the program. Table 3 details the requirements for Factors of Safety for Known (K_C) and Variable (K_V) components of Torque.

Table 3. Applied Factors of Safety to Torque Components per GEVS		
Program Phase	**Known Factor of Safety (K_C)**	**Variable Factor of Safety (K_V)**
Preliminary Design Review	2.00	4.0
Critical Design Review	1.50	3.0
Acceptance / Qualification Test	1.50	2.0

Variable torque components (K_v) are values that may vary from unit to unit or may increase over time, such as friction. Known torque components (K_c) are much more stable and cannot increase over time. Examples of these torque components are Motor Coulomb Torque and torque to accelerate inertias. The strict interpretation of the GEVS requires high safety factors at early stages of the program. These factors are intended to apply to new development efforts. Most times for actuator applications, characteristic frictions and performance are characterized on standard motor and gearbox frame sizes. For applications that have functionally tested or qualified components, it is entirely appropriate to use CDR level factors for early program margins, and use Acceptance / Qualification levels for the specification and final deliverable product.

One of the torque contributing components is accelerating the motor and load inertia at each step of the actuator. This component, reflected to the load is as follows:

$$T_{\alpha L} = \frac{J_M \cdot Kc \cdot N^2 \cdot \Delta\theta_A}{\Delta t^2} + \frac{J_L \cdot Kc \cdot \Delta\theta_A}{\Delta t^2 \cdot \eta_G} \quad (8)$$

Where Δt is the pulse time or 1/PPS.

Note: **$\Delta\theta_L$ must be in radians for this equation.** Additionally, gearbox efficiency is not applied to the motor inertia component because that torque is applied directly to the motor rotor. Motor acceleration torque is simply reflected to the output to provide consistent analysis.

Using the margins described above to determine the minimum required torque at Rated Velocity (PPS) of a geared stepper motor actuator, apply Equation 9, where F_L is the nominal friction at the load:

$$T_{PPS\_Min} = (F_L \cdot K_V) + (F_L(1-\eta_G)(K_V - 1)) + T_{\alpha L} + (N\,(K_V - 1)(F_{BM} + F_{BG})) + (N(K_C - 1)F_{CM}) \quad (9)$$

While there are many terms in this equation, it is simply summing the separate components that contribute to torque loads with their margin. For this equation, *only the margin terms that apply are added to the motor and gearbox bearing friction and gearbox efficiency*. Those losses have already been factored in the T_{PPS_0} term. Only the margins have been added here because taking the margin out at the T_{PPS_0} would lead to a mathematical linear assumption error that would apply insufficient margin in proportion to velocity. The calculation for the conservative linear approximation torque of our actuator may be completed with known information. Actuator output velocity (ω_A) and the Inertia Factored Response Rare (RR_{JF}) are in degrees per second.

$$T_{PPS_A} = \frac{(RR_{JF} - \omega_A) \cdot (T_{PPS\_0})}{RR_{JF}} \quad (10)$$

The Dynamic Margin of Safety (MoS) is calculated in Equation 11. Note: MoS must be greater than zero, for adequate torque margin.

$$MoS = \frac{T_{PPS\_A}}{T_{PPS\_Min}} - 1 \qquad (11)$$

Properly Specifying a Stepper Motor Actuator

Perhaps the most important aspect of selecting and integrating a geared stepper motor actuator into a system is properly defining and testing the product. This may sound obvious, but many specifications for these products fall short of the goal. A properly specified actuator should specify the components of loading and electro-magnetically generated torque components for the load (system) side as well as the actuator side. Using the example, Tables 4 and 5 delineate the components, values and tolerances.

Table 4. System Level Parameters (+25º C Unless Specified)

Parameter	Units	Symbol	Value	Tolerance	Margin Type
Load Inertia	kg-m^2	J_L	5.7E-04	Nominal	Kc
Load Friction	N-m	F_L	0.294	Nominal	Kv
Dynamic Velocity at Load	Degrees/sec	ω_L	90	Nominal	-
Maximum System Temperature	ºC	t_{max}	+50º	Max	-
Minimum System Temperature	ºC	t_{min}	-20º	Min	-
Maximum Supply Voltage	VDC	V_{max}	34	Max	-
Minimum Supply Voltage	VDC	V_{min}	24	Min	-
	Degrees/Step	$\Delta\theta_A$	1.5	Min	-
System Step Resolution	Radians/Step	$\Delta\theta_A$	0.02618	Min	-
Unpowered Holding Torque	N-m	T_{BD}	0.08	Min	-

Of course there are many other requirements that contribute to an aerospace quality component, but presenting the nominal system characteristics are the first step in deriving the requirements for the actuator that will drive that system. From the System Level Parameters, defined in Table 4, simulation is used to determine the requirements for the actuator, using the Torque Margin Requirements and methods discussed above. It is important to communicate the NOMINAL values for the torque contributing parameters because otherwise, margin will be applied on top of margin. When calculating worst-case conditions, an application may be over-powering the actuator unnecessarily, or forcing a larger actuator frame size than need be.

Table 5 delineates the derived Actuator parameters, broken down to the components and composite assembly level. As with the System Level Parameters, the contributing components of torque and applicable type of margin is utilized for those factors. Additionally, verification method using conventional (A) Analysis, (D) Demonstration, (I) Inspection, (S) Similarity and (T) Test is provided.

Table 5. Actuator Level Parameters (+25º C Unless Specified)

Parameter	Units	Symbol	Value	Tolerance	Margin Type	Verification Method
Motor Performance						
Motor Inertia	kg-m^2	J_M	7.06E-07	Nominal	-	A
Bearing Friction Torque (at -20ºC)	mN-m	f_{BM}	1.0	Max	Kv	D, S
Magnetic Coulomb Torque	mN-m	f_{CM}	5.0	Max	Kc	D, S
Composite Motor Friction Torques (**See Note 1**)	mN-m	f_{TM}	6.0	Max	-	T
Rated Pulse Rate	Pulses per sec	PPS	60	Nominal	-	A

Table 5. Actuator Level Parameters (+25º C Unless Specified)						
Parameter	**Units**	**Symbol**	**Value**	**Tolerance**	**Margin Type**	**Verification Method**
DC Resistance Per Phase	Ohms	Ω_{25}	50	±10%	-	T
Motor Inductance Per Phase	Henries	L	0.025	±10%	-	T
Torque Constant Per Phase	mN-m/Amp	K_T	240	Min	-	T
Back emf Constant Per Phase	V/Rad/sec	K_B	0.24	Nominal	-	T
Rated Minimum Current Per Phase **(See Note 3)**	ADC	Imin	0.436	Ref	-	A
Holding torque at **Rated** ADC Per Phase	mN-m	T_{H25}	0.135	Min	-	T
No Load Response Rate	PPS	RR_M	275	Min	-	T
	Deg/Sec	RR_M	8250	Min	-	T
Motor Winding (Simplex or Redundant)	-	-	Simplex	-	-	A, I
Gearbox Performance						
Gearbox Ratio	-	N	20	Nominal	-	A, D
Fore-driving Friction	mN-m	F_{BG}	2.0	Max	Kv	T
Gearbox Dynamic Efficiency	%	n_G	90	Min	Kc	D, S
Gearbox Torsional Stiffness	N-m/Rad	K_G	1000	Reference	-	D, S
Gearbox Backlash	Degrees	θ_{BL}	0.10	Max	-	I
Output Shaft Axial Play	mm/kg	θ_{BA}	0.05	Max		I
Output Shaft Radial Play (measured at shaft end)	mm/kg	θ_{BR}	0.05	Max		I
Composite Actuator Performance (Performance at Minimum Supply Voltage)						
No Load Response Rate	PPS	RR_A	275	Min	-	T
	Deg/sec	RR_A	412	Min	-	T
Rated Load Inertia for Dynamic Testing	$kg\text{-}m^2$	J_L	5.7E-04	Nominal	-	A
Response Rate with Rated Load Inertia	PPS	RR_{JFA}	185	Min	-	T
	Deg/sec	RR_{JFA}	277	Min		T
Resonant Frequency For Dynamic Testing **(See Note 4)**	Hz	Fn	90	Min	-	A
Torque to Accelerate at Rated PPS **(See Note 2 / Equation 8)**	N-m	$T_{\alpha L}$	0.139	Reference	Kc	A
Velocity at **Rated** Pulse Rate	Deg/sec	ω_A	90	Nominal	-	D
Pull-In Torque at **Rated** Pulse Rate **(See Equation 10)**	N-m	T_{PPS_A}	1.30	Min	-	T
Unpowered Back-driving Torque **(See Note 5)**	N-m	T_{BD}	0.09 / 0.18	Min / Max	-	T
Step Angle at Actuator Output	Degrees	$\Delta\theta_A$	1.5	Nominal	-	I

Notes on Table 5:

1. The Composite Motor Friction Torque is the only parameter that is testable in the final motor configuration. Bearing Friction Torque may be characterized with a preloaded set of bearings on a shaft with no magnets. Magnetic Coulomb Torque may be determined from subtracting the measured f_{BM} form the measured $f_{TM.}$
2. The Torque to accelerate the motor and load inertia at the required pulse rates was presented in Equation 8, above. This is provided as a reference value.
3. To calculate the **Rated** minimum current per phase divide the maximum Resistance at 25º C (Ω_{25_Max}) by the minimum Supply Voltage (V_{Min}). It is recommended to test the Holding Torque (T_{H25}) at this value.
4. It is highly recommended that the Test Set Resonant Frequency be calculated to be greater than the operational Pulse Rate (**Rated** PPS). See Equation 12.
5. Unpowered Back-driving Torque should be specified as a minimum when the actuator detent torque is used to maintain position during launch. A maximum value should be specified in all cases, to identify a potential high-spot that may impede performance or create a latent issue.
6. Any test verification with (S) Similarity, should be characterized on a representative unit. Customer determination if Similarity verification must be conducted on Flight Production unit.

Properly Testing a Stepper Motor Actuator

Since the performance of the actuator starts with the performance of the motor, testing the components of the actuator at each modular level, as indicated in Table 5, is highly recommended. These tests are intuitive and self-explanatory and not discussed in detail here. However, dynamic testing of the actuator assembly is vital. It is imperative to simulate the system load conditions as much as *practical*ly possible. It is also desirable to measure performance at the integrated system but measuring performance at the actuator level can avoid issues that may arise at a higher-level assembly. Some actuator suppliers believe that as long as back emf is tested, the actuator has been characterized. This is an over-simplified and potentially dangerous approach. Too many factors contribute to the dynamic performance of geared stepper motor actuation and the “test-as-you-fly” mantra flows down to the actuator level.

Figure 5 shows a typical geared actuator test set with simulated load inertia and performance torque transducer. A variant of this test configuration is advantageous. Rather than utilizing a loading magnetic particle brake, replace that component with a geared velocity control motor to back-drive the actuator. Employing torque transducers with integral position information is useful for position versus torque (or Torque-Theta) testing.

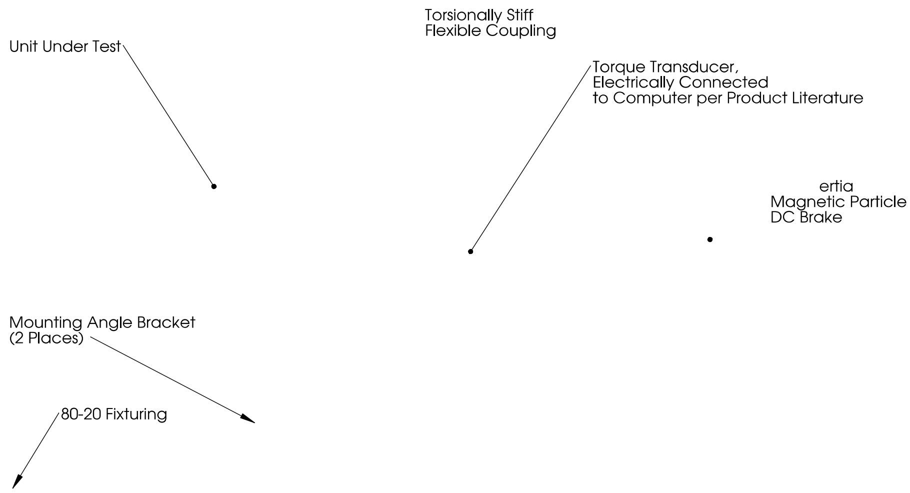

Figure 5. Example Stepper Motor Actuator Test Set-up

As stated in Table 5 and following notes, determining the resonant frequency of the test set-up is recommended. The Resonant Frequency (Fn), in Hz, of the test set-up is presented in Equation 12.

$$Fn = \frac{1}{2\pi}\sqrt{\frac{K_O(J_L+J_M \cdot N^2)}{J_L \cdot J_M \cdot N^2}} \qquad (12)$$

Where K_O is the overall system stiffness, which may be calculated by the taking the inverse of the sum of all the compliances in series:

$$\frac{1}{K_O} = \frac{1}{K_G} + \frac{1}{K_{FC1}} + \frac{1}{K_X} + \frac{1}{K_{FC2}} + \frac{1}{K_{MPB}} \qquad (13)$$

Where K_G is the gearbox torsional stiffness, and K_{FC1} is the stiffness of Flexible Coupling 1, K_x is the stiffness of the Torque Transducer, K_{FC2} is the stiffness of Flexible Coupling 2, and K_{MPB} is the stiffness of the Magnetic Particle Brake, all in N-m/rad. To achieve our desired load inertia of 5.4E-04 kg-m^2 it is necessary to hard couple a flywheel on the rear end of the magnetic particle brake. By connecting the simulated load inertia here, the compliance of the magnetic particle brake must be added into the equation. When stiffness components and inertia are factored into Equations 12 and 13, we obtain a resonant frequency (Fn) of over 220 Hz, well above the desired goal of 90 Hz. Minimum. Therefore it is known that resonances in the test set will not affect the dynamic test results at rated operational velocity.

Once the Unit Under Test (UUT) is fixtured to the test set, static and dynamic runout measurements should be made to assure alignments are proper. Also, there must not be any improper loading on the UUT or the test fixturing. It is recommended that No Load and Inertia Factored Response Rates be measured first. If there are any anomalous features of the UUT or the test set, these tests will highlight an issue quickly.

To check the Pull-In Torque, it is advised to set the magnetic particle brake to the minimum required dynamic torque value, or T_{PPS_Min}, and start the actuator from random rest position to verify the unit pulls-in at the specified pulse rate at minimum voltage. This is a Pass / Fail method that does not verify the actual magnitude of the pull-in torque, but rather that the minimum required value is achieved. To test the actual magnitude of the pull-in torque at a desired pulse rate is more of an iterative process. A relatively simple method that is not as laborious of continually increasing the torque test by test is to run the actuator at the desired pulse rate and increase the torque until the unit pulls-out of synchronous operation. While the unit is "buzzing" in this pulled out condition, decrease the brake torque until the actuator regains synchronous operation. Stop the unit and allow the actuator to return to room temperature, then test the pull-in torque at the torque value that returned the actuator to synchronous operation. A final minor adjustment may be necessary, but this value should be a negligible difference to the actual pull-in torque value.

Now that the performance at room temperature has been characterized, it may be desired to simulate or test the performance at maximum temperature. It is actually simple to simulate high temperature performance at ambient room temperature by calculating the motor resistance and power input as described in Equation 7 and Appendix A. With the reduced power calculated, simply adjust the power input to the supply to apply the high temperature input power, and duplicate the dynamic tests described above.

Adjusting the supply voltage cannot simulate testing an actuator at colder temperatures. While the electromagnetically generated torques are proportional in decreasing temperature, increased lube viscosity could dominate at colder temperatures and increase torque losses greater than the increased generated torque through reduced resistance. Testing inside a temperature chamber may be the only alternative, but if the cold temperature values are well within a lubricant's rating, room temperature performance may be acceptable.

Conclusion

Linear interpretation to simulate stepper motor performance when introduced to load inertia, given motor performance parameters, such as Motor Constant, Response Rate Constant and Motor frictional components has been presented. The analysis shows how to predict performance at room temperature as well as elevated temperatures. The linear approximation, however, does not predict actual variances (Cardinal Maxima and Minima) that naturally occur in stepper motor actuators. The magnitude and frequency of these variances that occur can be minimized through damping techniques.

The industry has applied the requirements of Torque Margin in many ways. To consolidate efforts and avoid over-margining, a breakdown of torque components and applied reasonable margins, in accordance with GSFC-STD-7000 has been presented.

Finally, recommendations for proper actuator specification and verification requirements were detailed. It is extremely important to conduct acceptance testing with a controller and test set-up that reflects the actual drive method and operational conditions that the actuator will see in the instrument.

References

1. Scott Starin "Stepper Motor Product Catalog and Design Guidelines V2.3" November, 2013
2. GSFC-STD-7000, "General Environmental Verification Standard" (GEVS), NASA Goddard Space Flight Center, Greenbelt, MD USA, 2005
3. Shane Brown, Scott Starin "Implications of Underdamped Stepper Mechanism Performance and Damping Solution Methodology", Proceedings of the 39th Aerospace Mechanism Symposium, NASA Marshall Space Flight Center, May 7 – 9, 2008

Appendix A
Stepper Motor Performance Equations (Two Phase Motor)

Parameter	Symbol	Units	Equation
Step Angle at Actuator Output	$\Delta\theta_A$	Degrees	$\Delta\theta_A = \frac{\Delta\theta_M}{N}$
Velocity at Actuator Output	ω_A	$\frac{Deg}{Sec}$	$\omega_A = \frac{\Delta\Theta_M * PPS}{N}$
Load Inertia Reflected to Motor	J_{LM}	kg-m^2	$J_{LM} = \frac{J_L}{N^2}$
Inertia Factor	J_F	-	$J_F = \frac{J_{LM} + J_M}{J_M}$
Total Power Input at Holding (at +25º C)	P_{H25}	Watts	$P_{H25} = \left[\frac{T_{H25}}{K_M}\right]^2$
Motor Constant (at +25º C)	K_M	$\frac{Nm}{\sqrt{Watt}}$	$K_M = \frac{T_{H25}}{\sqrt{P_{H25}}}$
Motor Constant (alternate equation)	K_M	$\frac{Nm}{\sqrt{Watt}}$	$K_M = \frac{K_T}{\sqrt{\Omega_{25}}}$
DC Resistance (at +25º C)	Ω_{25}	Ohms	$\Omega_{25} = \frac{2 \star V^2}{P_{H25}}$
DC Resistance at other Temperature (t_2)	Ω_{t2}	Ohms	$\Omega_{t2} = \Omega_{25}\ [1 + .004(t_2 - 25)]$
DC Current at Holding	I_{25}	Amps Per Phase	$I_{25} = \frac{V}{\Omega_{25}}$
DC Current at Holding at other Temperature (t_2)	I_{t2}	Amps Per Phase	$I_{t2} = \frac{V}{\Omega_{t2}}$
Total Power Input at Holding, at other Temperature (t_2)	P_{Ht2}	Watts	$P_{Ht2} = \frac{2V^2}{\Omega_{t2}}$
Torque Constant	K_T	$\frac{Nm}{Amp}$	$K_T = \frac{T_{H25}}{1.414 \cdot I_{25}}$
Torque Constant (alternate equation)	K_T	$\frac{Nm}{Amp}$	$K_T = K_M \cdot \sqrt{\Omega_{25}}$
Holding Torque at +25º C	T_{H25}	N-m	$T_{H25} = (N \cdot \eta_G) \cdot K_{M25} \cdot \sqrt{P_{H25}}$
Holding Torque at +25º C (alternate equation)	T_{H25}	N-m	$T_{H25} = (N \cdot \eta_G) \cdot K_T \cdot I_{25} \cdot 1.414$
Holding Torque at other Temperature (t_2)	T_{Ht2}	N-m	$T_{Ht2} = (N \cdot \eta_G) \cdot K_T \cdot I_{t2} \cdot 1.414$
Torque at Low Pulse Rate at +25ºC	$T_{PPS\text{-}0}$	N-m	$T_{PPS\_0} = (T_{H25} \cdot 0.707) - N(f_{BM} + f_{CM} + f_{BG})$
No Load Response Rate at Actuator Output (at +25º C)	RR	$\frac{Deg}{Sec}$	$RR = \frac{K_{RR} \cdot 6 \cdot \sqrt{P_{H25}}}{N}$

Appendix A
Stepper Motor Performance Equations (Two Phase Motor)

Parameter	Symbol	Units	Equation
Inertia Factored Response Rate at Motor (at +25º C)	RR_{JF}	$\frac{Deg}{Sec}$	$RR_{JF} = \frac{K_{RR} \cdot 6 \cdot \sqrt{P_{H25}}}{N * \sqrt{J_F}}$
Acceleration Torques, Reflected to Load **Note- $\Delta\theta_A$ must be in radians for this equation**	$T_{\alpha L}$	N-m	$T_{\alpha L} = \frac{J_M \cdot Kc \cdot N^2 \cdot \Delta\theta_A}{\Delta t^2} + \frac{J_L \cdot Kc \cdot \Delta\theta_A}{\Delta t^2 \cdot \eta_G}$
Available Pull In Torque at Required Pulse Rate at Actuator Output	T_{PPSA}	N-m	$T_{PPS_A} = \frac{(RR_{JF} - \omega_A)\ (T_{PPS\_0})}{RR_{JF}}$
Required Pull In Torque at Required Pulse Rate at Actuator Output (Interpreted from GSFC-STD-2000)	T_{PPS_Min}	N-m	$T_{PPS\_Min} = (F_L \cdot K_V) + (F_L(1 - \eta_G)(K_V - 1)) + T_{\alpha L} + (N\ (K_V - 1)(F_{BM} + F_{BG})) + (N(K_C - 1)F_{CM})$
Margin of Safety (must be >0 for adequate torque margin)	MoS	-	$MoS = \frac{T_{PPS\_A}}{T_{PPS\_Min}} - 1$
Natural Circular Resonant Frequency	Fn	Hertz	$Fn = \frac{1}{2\pi}\sqrt{\frac{K_o(J_L + J_M \cdot N^2)}{J_L \cdot J_M \cdot N^2}}$

Appendix A Variable Definition		
Symbol	**Units**	**Definition / Comment**
$\Delta\theta_M$	Degrees/step	Step Angle at Motor. Supplier provided value.
PPS	Steps Per Second	Application Dynamic Pulse Rate (Pulses Per Second)
N	-	Gearbox Ratio. Supplier provided value.
J_L	kg-m^2	Load Inertia
J_M	kg-m^2	Motor Inertia. Supplier provided value.
K_{RR}	$\frac{RPM}{\sqrt{Watt}}$	Motor Response Rate Constant – Supplier provided value. Value will change for drive method and whether unit is simplex or redundant.
η_G	%	Gearbox Efficiency. Supplier provided value.
f_{BM}	N-m	Motor Bearing Rolling Friction. Supplier provided value.
f_{CM}	N-m	Motor Coulomb or Detent Torque. Supplier provided value.
f_{BG}	N-m	Gearbox fore-driving bearing friction. Supplier provided value.
K_C	-	Torque Margin Factor for Known (Stable) Source. Value changes at maturity stage of the program. See Table 3.
K_V	-	Torque Margin Factor for Variable (Potentially Changing) Source. Value changes at maturity stage of the program. See Table 3.
F_L	N-m	Friction at the Load
K_O	N-m/Radian	System Overall Torsional Stiffness.
V	Volts, DC	Supply Voltage (should be de-rated for electronics headroom)
Δt	Seconds	Seconds per step or 1/PPS