Reliability Calculation Methodologies for Mechanisms and Actuators

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Abstract

MIL-HDBK-217 and similar reliability calculation methodologies inadequately assess the reliability of complex electro-mechanical components and mechanisms. This paper utilizes accepted reliability methodology techniques to achieve realistic and technically justifiable approaches for MTBF and reliability ratings. Additionally, this paper will address the historically overlooked effects of radiation on the accelerated degradation of insulation systems, such as those used in motors and other wound components.

Introduction

The challenging overall system requirements for aerospace mechanisms and gearboxes have led to several innovative and unique solutions for the application. One of the most demanding of system requirements is the reliability and number of steps. Even when the load friction is minimal, there is considerable transmitted torque through the gearbox to accelerate the load inertia. We will assess the methodologies used in determination of transmitted torques, as well as several other key requirements.

Analysis of Transmitted Torques

Calculation of Load Acceleration Torques

An often-overlooked implication of high torque margin in driving inertial loads is the acceleration torques at each step of the stepper motor. These torque pulses can be significant and must be considered in the mechanical structural and life analysis. For an example application, assume the load inertia is coupled directly to the output shaft, supported by a separate bearing system. Also, assuming the load friction is extremely low, and the driven load inertia is relatively high, each step of the stepper motor will accelerate the load and this will translate a reaction torque throughout the actuator. If the motor current is limited to maintain a maximum holding output torque this translates to a torque at low pulse rate ($T_{PPS\text{-}0}$) of at the motor. Using the torque at low pulse rate for these calculations will provide the *mean acceleration* and torque during each step. The Peak Torque and acceleration will be 41% higher than the mean calculated values.

For applications that do not implement current limiting, nominal and maximum motor torque calculations should be analyzed. That is, nominal voltage, resistance and temperature for the nominal case, and minimum temperature and resistance as well as maximum torque constant and voltage for the maximum case. The engineer needs to make sure that they have structural and endurance margin at maximum conditions, but it is also of value to assess the nominal conditions to gage how conservative the analysis is. Since torque margin requirements are calculated at the minimum conditional values, the maximum conditions may result in surprising results.

The mean acceleration at the load (α_L) is calculated in Eq. 1. The mathematical proof of this equation is available by contacting the authors.

$$\alpha_L = \frac{(T_{PPS-0} - F_M)(N \cdot \eta_G) - F_L}{J_L + (J_M \cdot N^2 \cdot \eta_G)} \tag{1}$$

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where (Application Values)

- α_L = Mean Acceleration of the Load
- $T_{PPS\text{-}0}$ = Torque at Low Pulse Rate at Motor
- F_M = Detent Plus Friction at the Motor
- N= Gear Ratio
- F_L = Load Friction
- J_L = Load Inertia
- J_M = Motor Rotor Inertia
- η_G = Gearbox Efficiency

The Torque to Accelerate the Load ($T_{\alpha L}$) is calculated by Eq. 2:

$$T_{\alpha L} = J_L \cdot \alpha_L \tag{2}$$

This torque is actually transmitted through the gearbox at every pulse of the system. As a note, since the gearbox efficiency attenuates the acceleration at the output, it is most conservative to use 100% gearbox efficiency.

Calculation of Loaded Time and Mean Loaded Velocity

Now that we have determined the peak-transmitted torque to the load, we must determine the *time* the unit is under load. Stepper motors do not transmit torque between step pulses when the shaft is settled. As long as the inertia factor is reasonable (under 3.0) the overshoot and stabilization torque are also insignificant. Therefore, we are primarily interested in the *time* it takes to accelerate the load at each pulse ($t_{\alpha L}$). Which is estimated in Eq. 3.

$$t_{\alpha L} = \sqrt{\frac{2 \cdot \Delta\Theta_L}{\alpha_L}} + \tau_e \tag{3}$$

where (Application Values)

- $t_{\alpha L}$ = Time to Accelerate Load
- $\Delta\Theta_L$ = Step Size at Load
- α_L = Acceleration at Load
- τ_e = Motor Electrical Time Constant

This results in a time to accelerate the load at each pulse of in a period of seconds. Given the life requirement of a number of steps a Loaded-Lifetime requirement of can be calculated. There are several offsetting secondary and tertiary components that affect the actual load characteristics and step kinematics, however, this analysis is considered conservative and appropriate. System drive electronics, internal damping characteristics as well as backlash will have minor effects on the step kinematics. These system variables can be difficult to predict or quantify, and their affects are much less significant compared to the primary variables defined in Eqs. 1-3.

Now that the torque and time components are established, we must determine the mean velocity when the mechanical energy is transferred. This calculation is presented in Eq. 4.

$$\omega_{L\_Loaded} = \frac{\Delta\Theta_L}{t_{\alpha L}} \tag{4}$$

where (Application Values)

- ω_{L_Loaded} = Mean Loaded Velocity

This analysis results in a Mean Loaded Velocity in Radians per second. The analyzed load acceleration torques ($T_{\alpha L}$), Mean Loaded Velocities, and Loaded Lifetime must be used in the gearbox and bearing endurance and analysis. The most important factor that we are communicating here is that *the torque and velocity that is transmitted through stepper motor actuators are typically significantly higher than the load friction and step-rate velocity requirements. Increasing torque motorization margin will necessarily increase the torque transmitted through the gearbox, regardless of the magnitude of the load frictional components.*

Another key take-away, the Loaded Lifetime can be orders of magnitude less than the operational time, since stepper motor actuators are not transmitting torque during idle and settled times between steps. Additionally, if you run through the numbers, you will see that lower gear ratios will reduce $T_{\alpha L}$ at the actuator output. Doing this will increase your inertia factor and increase the probability of unstable step performance, as described in Ref. [1].

Reliability Analysis

There are many statistical approaches to calculate the failure rates of components and systems. Many of which do not adequately assess the failure rates of electro-mechanical systems such as a mechanical. It is also impractical to manufacture a statistically significant number of units to determine failure rates and probabilities. The authors believe the approach presented herein is much more appropriate and statistically valid for mechanisms and components manufactured for the space industry.

Reliability Block Diagram

The example in Fig. 1 breaks down the components of an actuator in a Reliability Block Diagram. The motor may be driven through either the primary or redundant motor winding. Single-string gearbox components are typical for the industry, as differential gearing adds significant cost, mass and complexity. The output position may be measured by either the primary or redundant sensor windings. Reliability Block Diagrams may seem trivial, but it provides a graphic representation of the system configuration and aides in the determination of potential failure modes and their effects. Also of note in the example that the mechanism is still functional even with the loss of both resolvers. Step counting or optical performance data may be used to determine load position.

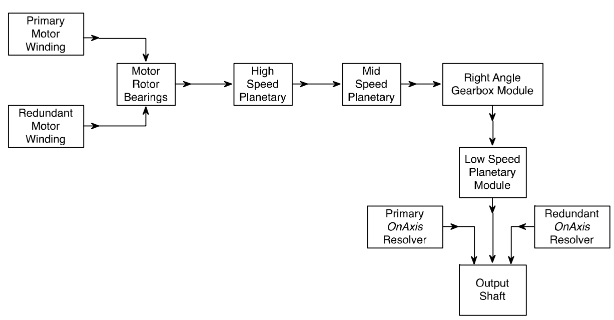

Figure 1. Reliability Block Diagram

Reliability Analysis of Each Component

The methodologies of the reliability of the components are presented in the 3.3.1 of this paper. The detail summaries are shown in Appendix A herein. This methodology uses a combination of estimated L_2 life analysis, combined with classical Weibull reliability statistical analysis. We must first determine the L_2 of the Mechanical and Electrical components of the actuator. Note: Avior uses L_2 for most space flight hardware analysis, although mathematically, you will achieve the similar reliability figures for Characteristic Life and MTBF if you use L_{10} values.

Gears and Bearings

Structural bending fatigue analysis is conducted on all gears, per AGMA 2001. The analysis is conducted to determine how many cycles and hours of operation at the mean torque and velocities described in section 3 herein may be achieved. We calculate the L_2 life each of the mechanical gearing elements in the gearbox, using the AGMA equations. This calculated value tabulated in the Appendix A table. The authors conservatively use one million hours of L_2 life for each module if the calculated value is greater than one million hours.

Bearings are similarly calculated for a factored L_2 life using classical tools and software. In the right-angle gearbox, the bearings see thrust, tangential, and radial loads due to the bevel gear mesh. If there are pure-torque modules in the actuator, conservative estimates of radial loads are provided, due to imperfect gear meshing and load sharing in the planetary gearboxes. Typically, 10 to 50 Newtons of radial force at the extreme of the gear mesh is assumed for the supporting bearing loads. Planet gear bearings see radial loads through the transmitted torque at each stage of gearing.

Brancato Method of Motor Winding Life

Winding life estimates are analyzed using methods described in the Brancato Method, Ref. [3]. Avior's Class H220 Insulation system is rated at 20,000 hours minimum regression life at +220º C. The Percentage of Life at a conservatively estimated Hot Spot Temperature of +120º C is as follows:

$$Lw = 100 \cdot 2^{\frac{T_R - T_{HS\_M}}{10}} \tag{5}$$

where

- L_W= Percentage of Life (To Be Calculated)
- T_R = Rated Insulation System (+220º C)
- T_{HS_M} = Motor Winding Hot-Spot Temperature (+120º C)

This equates to a lifetime of **102,400 percent greater** than a typical minimum regression rating of 20,000 hours. This translates to 2.05 E+07 Hours of life for the winding insulation system. In this application, Avior generally uses an attenuation factor of for motor and sensor windings used in Geosynchronous and Deep-Space (high radiation) applications. In the next section, we will introduce a more appropriate de-rating strategy. When exposed to gamma ray and ultraviolet radiation, there are insulation aging degradations that are similar in effects to thermal radiation aging. Due to ultraviolet radiation having low penetration it can be neglected. The following section introduces alternate methods for dealing with the effect of ionizing radiation aging of electrical components.

Radiation De-rating of Winding Assemblies

As denoted above, and for the purpose of derating, it is assumed that geosynchronous radiation similarly degrades electrical components as thermal radiation. Two methods based on similar principles were developed to handle the effects of ionizing radiation aging. The first method (ψ) accounts for both the amount of radiation absorbed and the rate at which the radiation is absorbed. The second (γ) incorporates the amount of radiation absorbed.

The first method is to calculate an effective temperature from the incident radiation and ambient temperature, with the components approximated as a black body. Thermal radiation is taken into account with T_a.

$$T_{eff} = \sqrt[4]{T_a^4 + \psi * \zeta} \tag{6}$$

where

- T_a = ambient or operating temperature (K) T
- ζ = incident space radiation / σ_B (space radiation is about 7 W/m^2)
- σ_B = Stefan-Boltzmann constant 5.67E-08
- ψ = Harrington Psi Life Function (proposed), de-rates electrical and insulation components

$$\psi(\delta) = a * \delta^b \tag{7}$$

where

- δ = Total ionizing dose of space radiation during the mission (rad)
- a = proposed constant, for Teflon insulation applications, a = 1.176

- b = proposed constant, for Teflon insulation applications, b = 0.144

The constant in Eq. 7 arises from the success of the chosen insulation system. The equation is used to add an additional margin of safety to the degradation of electrical components. Introducing ε the exposure ratio,

$$\varepsilon = \frac{L_o}{L_M} \tag{8}$$

where
- L_o = Electrical Operating Time in Hours
- L_M = Mission Life in Hours

Finally, combing the results into the Brancato Method:

$$L_w = 100 * (\varepsilon * 2^{\frac{T_R - T_{effo}}{10}} 10 + (1-\varepsilon) * 2^{\frac{T_R - T_{effi}}{10}}) \tag{9}$$

where
- L_w and T_R can be found following Eqn. 5

One alternative to energy methods for calculating insulation life due to radiation is to assume that the insulation follows the reliability equation. Instead, we propose time be replaced with total ionizing dose (rads)

$$R = e^{-(\frac{\delta}{\eta})^{1.2}} \tag{10}$$

where
- δ = Total ionizing dose of space radiation during the mission (rad), assumed to be 10000 rads for 5-year mission at geosynchronous orbit
- η = 4.25E7 rads, proposed constant for insulation reliability, see appendix C

Reliability Analysis Toolkit

Using the on-line tools of the Reliability Analysis Toolkit, Ref. [2], we are able to predict the reliability failure rate of an actuator. The methods used in this approach are derived from the NSWC Handbook of Reliability Prediction Procedures for Mechanical Equipment. The online tools, using the equations described in 3.3.2 applicable Weibull distribution functions for different components.

Reliability Equations

Using the L_2 to MTBF Conversion, the equations represent a three-parameter Weibull distribution, with δ shown in the equations; however, for purposes of this tool this parameter (sometimes called "failure free life") is assumed to be zero, and eliminated from these equations. Note, the variables and symbols used in this section are consistent with those used in Ref. [2].

Reliability Function R(t):

$$R(t) = e^{-\left(\frac{t}{n}\right)^{\beta}} \tag{11}$$

Probability Density Function f(t):

$$f(t) = \frac{\beta(t)^{\beta-1}}{\eta^{\beta}} e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{12}$$

h(t), instantaneous failure rate:

$$h(t) = \frac{f(t)}{R(t)} \tag{13}$$

The average failure rate is calculated using the Eq. 14:

$$\lambda_{Average} = \frac{1 - R(T)}{\int_0^T R(t)\,dt} \tag{14}$$

System Input Parameters:

1. Shape parameter (β): Weibull shape “Beta” factors. From Ref. [2]. Usages of Typical or High-End factors are provided. A Beta factor of 1.2 is used for windings, 1.3 for Ball Bearings and 2.0 for Gearboxes. Higher complexity assemblies utilize higher β factors.

2. Characteristic Life (η) is the 63.2% failure point for a mechanical system. The Characteristic Life is determined by the L_2 life and the β factor. For instance, for a β of 1.3, as for a bearing, the Characteristic Life is about 20x the L_2 life. For a β factor of 2.0, the multiplication factor is about 7x the L_2 life. η can be calculated exactly by using Eq. 15.

$$\eta = L_n \left(-ln \left(1 - \frac{n}{100} \right) \right)^{-1/\beta} \tag{15}$$

3. Maintenance interval for item renewal (T): No maintenance of this product is required or assumed. A value of 100 years is used in the on-line tool equations.

<u>MTBF with the Reliability Toolkit</u>

The on-line Reliability Toolkit was used to generate a MTBF, from the β function and the L_2 predicted life. The same equations used in the on-line tool are available in this paper. The lifetime estimations of the bearings and gears are based on conservative de-ratings of fatigue and wear-out considerations, with margin, the failure rate of these estimations will be proportional to the required life over the predicted life. This method has been used in reliability analysis presented to, and accepted by many other programs and subject matter experts in the industry. The justification for this analysis methodology is the additional manufacturing precision, workmanship testing and quality inspections that take place for space flight hardware to eliminate infant mortality and other workmanship issues that affect commercial hardware.

MTBF can be calculated using Eq. 16

$$MTBF = \frac{\frac{\eta}{\beta}\gamma\left(\frac{1}{\beta}, \left(\frac{T}{\eta}\right)^{\beta}\right)}{1 - e^{-\left(\frac{T}{\eta}\right)^{\beta}}} \tag{16}$$

where, γ(a,x) is the lower incomplete gamma function.

Equation 11 can be ‘altered’ by setting MTBF = T. By doing so, a ‘Maintenance Free’ interval is achieved i.e. the part on average would fail at the maintenance interval. The equation reduces to η/β*Γ(1/β) for ‘No Maintenance. See Appendix C for the method and code.

<u>Redundancy Factor for Windings</u>

Since there is selectable redundancy for the motor and resolver windings, we may use the Redundancy Factor described in the Reliability Toolkit. We use the Effective Failure Rate on One Standby Offline Unit with One Active On-line Unit Required for Success (without repair), See Eq. 20.

<u>Reliability Number</u>

We essentially have two separate life analyses to consider, the mechanical reliability and the electrical reliability. The windings and the electrical insulation system are exposed to radiation the entire time in orbit, so regardless of powered-on time we assume the time on orbit degrades the insulation system. For a five-year orbital mission for a high-radiation, geosynchronous application, we have an electrical life requirement

(t_E) of 44,000 hours of operation and 2E+04 Rads Total Ionizing Dose (TID) of radiation during mission life. Refer to Appendix A for a table of analysis parameters.

To calculate the Mechanical Reliability for the operational profile from the MTBF, Eq. 17 is used.

$$R_{tM} = e^{\left[\frac{-t_M}{MTBF_M}\right]} \tag{17}$$

where
- R_{tM} = Mechanical Reliability
- t_M = Time of loaded operation (Hours)
- $MTBF_M$ = Mechanical Mean Time Between Failures (1.20E+05)

This results in an R_{tM} = 0.99736
Similarly, the Electrical Reliability is calculated by:

$$R_{tE} = e^{\left[\frac{-t_E}{MTBF_E}\right]} \tag{18}$$

where
- R_{tE} = Electrical Reliability
- t_E = Time of exposure, in Hours (44,000)
- $MTBF_E$ = Electrical Mean Time Between Failures using (8.95E+12) calculated using γ de-rated Averaged Brancato method.

Which results in an R_{tE} =0.99999

The insulation reliability is calculated in equation (19)

$$R_i = e^{\left[\frac{-\delta}{MTBF}\right]} \tag{19}$$

where
- R_{tM} = Insulation Reliability
- δ = Total Ionizing Dose (10000 rads)
- $MTBF$ = Insulation Mean Time Between Failures (4E+7)

Which results in an R_i =.99975.

And the overall reliability (R_O) is calculated by:

$$R_o = R_M \cdot R_E \cdot R_i \tag{20}$$

Therefore, the overall reliability *Ro* = 0.99701 is achieved.

The MTBF for n redundant units, 1 spare with no repair interval:

$$MTBF_R = MTBF * n * (P + 1) \tag{21}$$

where
- n = number of active units
- P the probability that the spare will work

The probability of the spare working can be treated as that units' reliability. Using these equations for both cases of the electrical components results in *Ro* = .99736 for the ψ factored electrical decay. See Appendices A and B for more information on reliability values. If we needed to increase the resultant overall reliability, we would address the two lowest MTBF components. In this case, the Mid-Speed Carrier

Bearings that see the radial and thrust loads from the bevel mesh and the low-speed planetary module are the reliability-driving components. Possible methods to increase reliability in these components is to increase the gearbox and bearing size, or possibly change materials to increase the L_2 ratings.

Conclusion

Requirements for torque margin may have structural and endurance implications for transmitted torque through the mechanism that must be considered. Non-current limited applications may also have significant torque consequences at extreme conditions that are often not analysed. The paper also details a methodology for calculating reliability using approved analysis techniques and methodologies. We also introduce two types of analysis for de-rating life estimates for motor insulation systems, due to insulation aging effects due to prolonged exposure to radiation.

References

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Appendix A – MTBF and Reliability Table - Mechanical

A	B	C	D	E	F	G
Subsystem Module or Component	Beta (Weibull Shape Parameter)	L_2 Life (Hours)	Characteristic Life (Hours)	MTBF (Hours)	Average Failure Rate (Hours)	Reliability
N/A	β	See 5.2	η	Ref. [2]	λ	Eq. 5
Motor Rotor Bearing	1.3	4.50E+05	9.05E+06	9.57E+06	1.05E-07	0.99997
High Speed Planetary Module	2	1.00E+06	7.04E+06	7.79E+06	1.28E-07	0.99996
High Speed Planet Gear Bearings	1.3	1.00E+05	2.01E+06	2.13E+06	4.70E-07	0.99985
High Speed Carrier Bearings	1.3	5.27E+05	1.06E+07	1.12E+07	8.93E-08	0.99997
Mid Speed Planetary Module	2	1.00E+06	7.07E+06	7.79E+06	1.28E-07	0.99996
Mid Speed Planet Gear Bearings	1.3	8.75E+05	1.76E+07	1.86E+07	5.38E-08	0.99998
Mid Speed Carrier Bearing	1.3	2.94E+04	5.91E+06	6.25E+05	1.60E-06	0.99949
Right Angle Bevel Gears	2	1.00E+06	7.04E+06	7.79E+06	1.28E-07	0.99996
Right Angle Carrier Bearing	1.3	3.17E+07	6.38E+08	6.74E+08	1.48E-09	0.99999
Right Angle Support Bearing	1.3	5.40E+06	1.09E+08	1.15E+08	8.71E-09	0.99999
Low Speed Planetary Gearing	2	2.50E+04	1.76E+05	1.95E+05	5.14E-06	0.99837
Low Speed Planet Gear Bearings	1.3	1.03E+05	2.07E+06	2.19E+06	4.57E-07	0.99985
Low Speed Carrier Bearings	1.3	2.34E+06	4.71E+07	4.97E+07	2.01E-08	0.99999
			Total	1.20E+05	8.33E-06	0.99736

Appendix B – MTBF and Reliability Table - Electrical							
A	B	C	D	E	G	H	I
Subsystem Module or Component	Beta (Weibull Shape Parameter)	L_2 Life (Hours)	Characteristic Life (Hours)	MTBF (Hours)	Reliability	R_o	Redundant R_o
.1Avior standard* at te=44000	1.2	2048000	5.29E+07	2.76E+07	0.99841	0.99577	0.99656
.1 Averaged Brancato at te=44000	1.2	2.60E+09	2.69E+10	1.40E+10	0.99999	0.99736	0.99736
ψ at te=44000	1.2	1.47E+11	3.80E+12	1.98E+12	0.99999	0.99736	0.99736
γ Averaged Brancato at te=44000	1.2	6.63E+11	1.13E+13	8.95E+12	0.99999	0.99736	0.99736

Calculations of reliability and MTBF include both the On-Axis Resolver and Motor Windings
*The Avior "old" standard is the Brancato L_2 life at 120˚C multiplied by .1
The averaged Brancato method can be found in Eq. 9 and in Appendix D
A value of -30˚C was used for the cold or non-powered state of electrical components
A value of +120˚C was used for the hotspot temperature of electrical components

Appendix C

The following code was written in Anaconda Spyder to calculate η, MTBF, λ, and no maintenance interval (T)

```
# denotes code comment
from scipy.special import gammaincc, gamma
from math import *
from math import exp as e
B = β                           #beta shape parameter
L2 =L₂                          #L2 life
n = L2*(-log(.98))**(-1/B)      #η, characteristic life for L2
le = 5*365*24                   #mission life for electrical components (Hours)
lm = 318                        #usage life requirement for
T = 50                          #initial guess in hours of no maintenance interval
R = e(-(T/n)**B)                #reliability function
MTBF = (n/B*gamma(1/B)*gammainc(1/B,(T/n)**B))/(1-R)     #see (◊)
err = abs(T-MTBF)/MTBF*100      #percent error
ep = .1                         #final percent error
while err>ep:                   # subroutine solves T for conditions stated after Eq. 11 by simply convergence
   T = MTBF                     # Iterated map on T
   R = e(-(T/n)**B)
   MTBF = (n/B*gamma(1/B)*gammainc(1/B,(T/n)**B))/(1-R)
   err = abs(T-MTBF)/MTBF*100
print(n,MTBF,1/MTBF,e (-lm/MTBF))      #reports results η, MTBF, λ, and reliability Rn
print(T/365/24,"Years")                #no maintenance life in years
```

◊scipy gammainc is normalized, so it has to be multiplied by gamma

```
from scipy.special import gamma
B = β                           #beta shape parameter
L2 =L₂                          #L2 life
n = L2*(-log(.98))**(-1/B)      #η, characteristic life for L2
R = e(-(T/n)**B)                #reliability function
MTBF = n/B*gamma(1/B)
```

Appendix D

Graph of ψ implemented Life decay, implementing Eq. 9

Averaged Broncato Equation (Generalized Equation 9),

$$L_w = 100 * \sum_i \varepsilon_i * 2^{\frac{T_R - T_i}{10}}$$

where

$$\sum_i \varepsilon_i = 1$$

- T_i = some temperature
- ε_i =percent exposure to T