

DESIGN AND MANUFACTURING CONSIDERATIONS FOR MODIFIED INVOLUTE GEARING

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ABSTRACT

This paper provides an introductory overview of design and manufacturing considerations for modified involute gearing, with a focus on space applications. Depending on the requirements for the gear train, various degrees of modification, including profile modification may be necessary to ensure smooth-running operation and long life. Full consideration of manufacturability will result in a higher AGMA (2000) quality part. Manufacturability using conventional tooling is important to avoid excessive cost and lead time.

INTRODUCTION

Involute Profile

As shown in Figure 1, The involute profile is the curve traced by a point on a string while being tangentially unwound from a cylinder. In this example, the diameter of the cylinder is referred to as the “base diameter”.

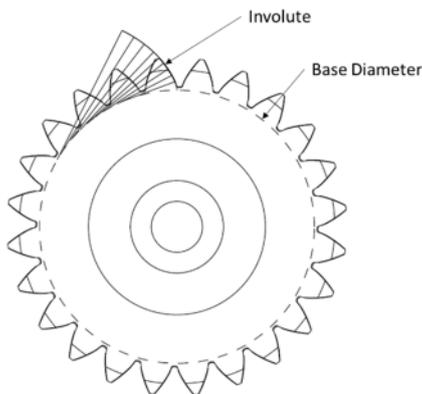


Figure 1: Involute Profile and Base Circle

Not surprisingly, as two gears mesh with each other, the point(s) of contact between gear teeth will move along the tangent line between the two base diameters, as shown in Figure 2. The pressure angle, which will typically range from 20 to 30 degrees, is the angle of this tangent line from the horizontal, also indicated in Figure 2. Note that gears with smaller contact angles use the lower portion of the involute curve, and gears with higher contact angles use a higher portion of the involute curve.

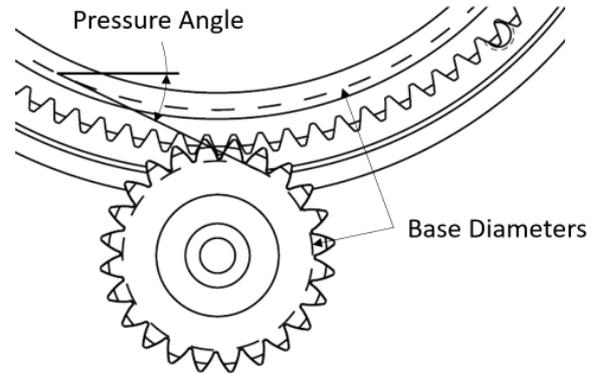


Figure 2: Pressure Angle and Tangent Line Between Base Diameters

The involute equation is simple and forms the basis of the kinematic calculations required for gear design. See Figure 3 and Eqs. (1) & (2).

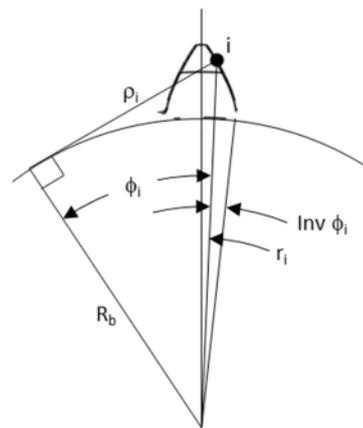


Figure 3: Involute Equation Variables Definition [1]

$$\phi_i = \cos^{-1} \left(\frac{R_b}{r_i} \right) \quad (1)$$

$$\text{inv} \phi_i = \tan \phi_i - \phi_i \quad (2)$$

Contact Ratio, Addendum, Dedendum

The length of the common tangent line which passes through the intersection between the two gears is called the “line of action”. The length of the line of action is used to determine another important parameter, the

contact ratio. The contact ratio is defined as the length of the line of action divided by the base pitch, per Eqs. (3) & (4) [1] and gives the average number of teeth in contact while the gears are turning. For smooth action, the contact ratio must be greater than 1.

External gears:

$$m_p = \frac{(r_o^2 - r_b^2)^{1/2} + (R_o^2 - R_b^2)^{1/2} - C_{op} \cdot \sin \phi_{op}}{p_b} \quad (3)$$

Internal gears:

$$m_p = \frac{(r_o^2 - r_b^2)^{1/2} - (R_i^2 - R_b^2)^{1/2} + C_{op} \cdot \sin \phi_{op}}{p_b} \quad (4)$$

Where:

- m_p = contact ratio
- r_o = effective outer radius of pinion
- r_b = base radius of pinion
- R_o = effective outer radius of gear
- R_i = effective inner radius of gear
- R_b = base radius of gear
- C_{op} = operating center distance
- ϕ_{op} = operating pressure angle
- p_b = base pitch

Contact ratio can be increased by increasing the addendum, which is the height of the teeth above the pitch radius. For external gears, the addendum height is the outer radius minus the pitch radius, and for internal gears it is the pitch radius minus the inner radius. Similarly, the dedendum is the height of the tooth below the pitch radius. For external gears this is the pitch radius minus the root radius. In this paper, addendum and dedendum heights are normalized by multiplying them by the diametral pitch, and expressed as dimensionless parameters.

Working Pitch Diameter and Operating Pressure Angle

The pitch diameter of a gear may be thought to be fixed by the gear design. There is, however, a working pitch diameter which is likely to differ from the design pitch diameter. For the purpose of defining and measuring a gear, the pitch diameter is simply the number of teeth divided by the diametral pitch. However, when two gears mesh, the effective pitch diameters are set by the center distance between the two gears. The center distance will always have some tolerance error and may even be adjusted or modified as described later. Therefore, a working pitch diameter must be considered. The working pitch radii can be defined per Figure 4 as the distance from the axis of each gear to the point on the line of action intersecting a line that passes through both gear axes. Similarly, a working pressure angle is also calculated based on the base diameters and working center distance.

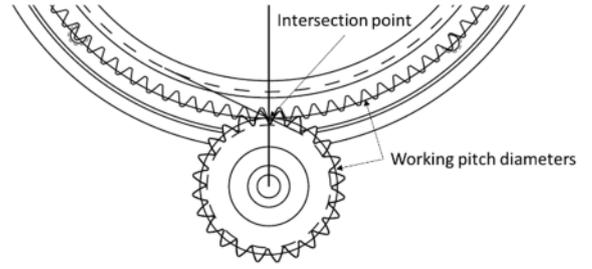


Figure 4: Working Pitch Diameters

$$\phi_{op} = \cos^{-1} \left(\frac{r_b + R_b}{C_{op}} \right) \quad (5)$$

$$r_{pw} = \frac{r_b}{\cos \phi_{op}} \quad (6)$$

$$R_{pw} = \frac{R_b}{\cos \phi_{op}} \quad (7)$$

Where:

- r_{pw} = working pitch radius of pinion
- R_{pw} = working pitch radius of gear

Specific Sliding

If we examine the velocities of each gear along the line of action, it should be noted that the direction and magnitude change along the line of action. Per Figure 5, The velocity at the contact point of the driving gear at the beginning of the mesh is tilted slightly upward while the velocity of the contact point on the driven gear is tilted slightly downward, indicating that there is a relative sliding velocity between the two at this point. The only location along the line of action where the two velocity vectors match is at the intersection of the line of action with the working pitch diameters. Here the sliding velocity is zero. This is referred to as pure rolling action. Except at this one point on the line of action, there is always some combination of rolling and sliding action. As can be seen in the figure, the mismatch in the velocity vector directions (or sliding component) increases the further the contact point is from the working pitch diameter.

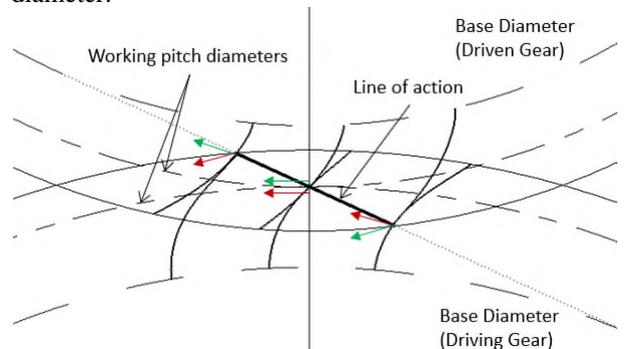


Figure 5: Velocity vectors along mesh contact: start of mesh, working pitch diameter, and end of mesh. Red vectors correspond to driving gear and green correspond to driven gear.

To calculate gear mesh efficiency, and as a metric for comparing gear designs, the ratio between sliding and rolling can be calculated. As shown in Figure 5 & Eq. (8), per [2], the sliding velocity is defined as the difference between the velocities of each gear at the point of contact. Per [2], the specific sliding can be expressed per Eqs. (11) & (12). Figure 6 shows a plot of specific sliding versus gear mesh angle.

$$\vec{v}_s = \vec{v}_{green} - \vec{v}_{red} \quad (8)$$

$$\rho_p = \sqrt{r_{pw}^2 - r_b^2} \quad (9)$$

$$\rho_g = C_{op} \cdot \sin\phi_{op} - \rho_p \quad (10)$$

$$v_{ssp} = 1 - \frac{\rho_p \cdot N_p}{\rho_p \cdot N_g} \quad (11)$$

$$v_{ssg} = 1 - \frac{\rho_p \cdot N_g}{\rho_g \cdot N_p} \quad (12)$$

Where:

- v_s = sliding velocity (vector)
- ρ_p = radius of curvature of pinion flank at contact point
- ρ_g = radius of curvature of gear flank at contact point
- N_p = number of teeth, pinion
- N_g = number of teeth, gear
- v_{ssp} = specific sliding, pinion
- v_{ssg} = specific sliding, gear

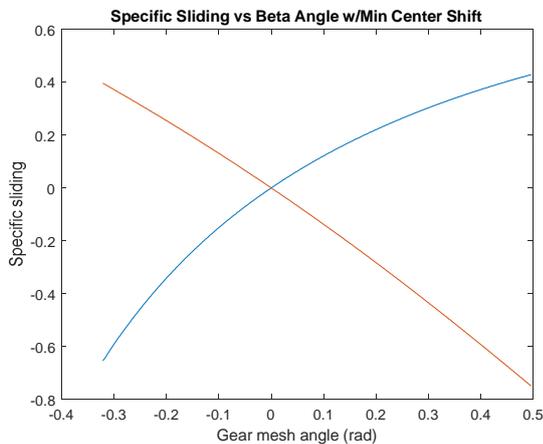


Figure 6: Specific Sliding Versus Gear Mesh Angle

Basic Gear Mesh Efficiency

Per [1], basic gear mesh efficiency (due to friction losses alone) can be calculated per Eq. (13). Note that the calculation is a function of specific sliding, operating pressure angle, and friction coefficient. Bearing drag torque, viscous losses, deflection, and other factors may

need to be accounted for to get the “real” efficiency of the gear system, depending on the application and gear arrangement. But it should be noted that while friction coefficients may be set by the lubrication and materials used, specific sliding and contact angle can be adjusted by the designer to improve efficiency.

$$E = 100 - \frac{50 \cdot \mu}{\cos\phi_{op}} \cdot \left(\frac{H_s^2 + H_t^2}{H_s + H_t} \right) \quad (13)$$

Where:

- E = gear mesh efficiency (%)
- μ = friction coefficient
- H_s = Specific sliding at start of approach action (Use Eq. 9)
- H_t = specific sliding at end of recess action (use Eq. 9)

LIMITATIONS OF CONVENTIONAL GEARING

Conventional, or unmodified gearing can be used in many applications. However, when it is desired to minimize specific sliding, whether to improve life or maximize efficiency, or when gears with a low tooth count are required, which can cause undercut teeth, a variety of methods can be used to modify the shape of the teeth while maintaining an involute profile that is still manufacturable using conventional tooling.

In applications with very high tooth loads, or when smooth operation is critical, modification of the involute profile itself may be necessary. In these applications, ground teeth are required to meet the fine tolerances necessary for such applications, enabling more flexibility and precision to accommodate such profile modification.

Undercutting

Per [1], when the number of teeth is small, (less than 17 for a pressure angle of 20°, depending on whole tooth depth), the cutter will interfere with the tooth flanks, causing undercutting as shown in Figure 7. Undercutting has several undesirable effects. First, it reduces the effective tooth width and bending strength, and thus reduces bending fatigue life.

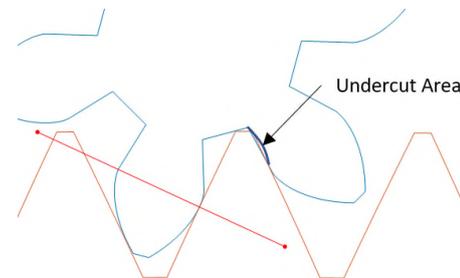


Figure 7: Undercut Tooth due to Tip-to-Flank Interference Between Hob (Orange) and Part (Blue)

Undercutting also reduces the effective contact ratio, as the tooth flank no longer extends down to the root fillet, which can cause noise. Finally, the abrupt transition to the undercut area can create a contact stress concentration, should the mating tooth make contact in this area. Fortunately, undercutting can be reduced or eliminated by various methods of tooth modification. [1] provides tables of minimum tooth counts to avoid undercutting for unmodified gearing. Once modification is used, if the minor diameter can be maintained larger than the base diameter, undercutting is not likely to occur.

Imbalanced Bending Strength

As shown in Figure 8, designs with only addendum and/or dedendum modification can create significant width difference between the teeth on the driving gear, and the teeth of the driven gear. This can lead to a reduced load capacity, or reduced bending fatigue life, but can be easily eliminated with tooth width correction.

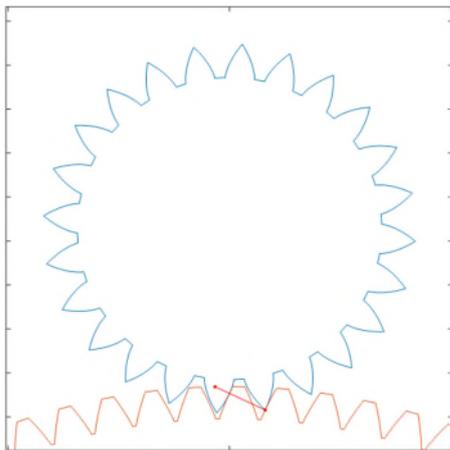


Figure 8: Gear Mesh with Long & Short Addendums with Uneven Tooth Widths

Specific Sliding

If the designer is limited to conventional gearing with unmodified teeth, the resulting design may suffer from reduced efficiency or even reduced wear life due to high or imbalanced specific sliding. It should also be noted that for maximum wear life, smooth action, etc. [3] advocates biasing toward recess action, which further necessitates tooth modification in the form of addendum/dedendum adjustment. Note that recess action is preferred since it reduces the maximum specific sliding as the slope of the specific sliding plot levels off in the recess action region (See Figure 6).

High Loads and Smooth Operation

In highly loaded or high precision applications where smoothness under load is critical, unmodified gearing will create additional vibration and gear errors in portions

of the contact where more than one tooth is in mesh. As shown in Figure 9, a tooth under load deflects. As the next tooth pair approaches to make contact, they are no longer aligned due to the deflection of the tooth already in contact. When they come together, they will crash, and suddenly pick up load, causing vibration and angular error. In these applications it is necessary to modify the involute profile itself to avoid such issues.

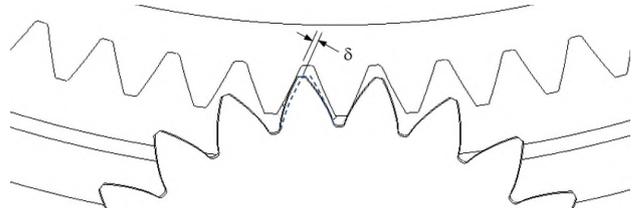


Figure 9: Tooth Deflection Under Load

Angular Misalignment

If angular misalignment between gear rotation axes occurs, whether from mounting tolerances or deflections under load, the tooth loading can become highly uneven and lead to excessive contact stress, and possibly excessive bending stress if the face width is large. One approach to eliminate this issue is to modify the profile longitudinally, via crowning of the gear teeth.

MODIFIED GEARING

Four methods for gear modification are covered in this paper: addendum/dedendum modification, tooth width correction, center distance modification, and profile modification.

Addendum/Dedendum Modification and Width Correction

Adjusting the addendum and dedendum heights is a very common method to eliminate or reduce undercutting, reduce specific sliding, reduce approach action and increase recess action, and generally optimize the gear design. In some cases, addendum/dedendum modification can lead to very narrow teeth on the driving or driven gear, and tooth width correction may be required to achieve the desired addendum amount. Per [1], the width of the tooth tip should be greater than or equal to 0.3 divided by the diametral pitch. Without widening the tooth, this places a practical upper limit on the addendum height. As the addendum height is increased on one gear, it must be decreased on the mating gear, in order to keep the tooth whole depth within the manufacturable range while avoiding tip-to-root interference. This give-and-take also biases the mesh toward recess action, which is desired. As shown in Figure 10, as the addendum of the driving gear is increased, while decreasing the addendum of the driven gear, the tooth widths become unbalanced, the driving gear develops a knife-edge at the tip, and ultimately reaches its limit for addendum modification. Conversely,

Figure 11 shows this same progression, while employing width correction to provide balanced tooth widths and workable designs.

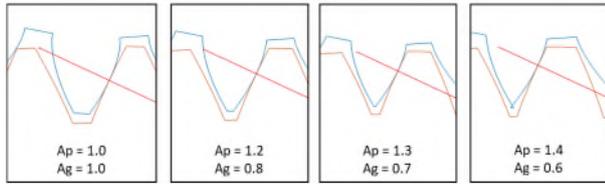


Figure 10: Tooth Profile versus Addendum without Width Correction

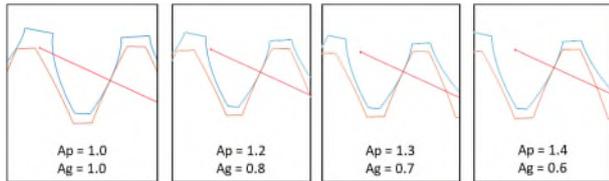


Figure 11: Tooth Profile versus Addendum with Width Correction

To eliminate undercutting, the addendum can be increased while decreasing the dedendum, until undercutting is eliminated. Tooth width correction may also be required.

Optimization of specific sliding and biasing toward recess action and/or maximizing efficiency can be accomplished with addendum and dedendum modification and tooth width correction. In this case, a design example will be used.

The kinematic design parameters for a small gimbal were to have a 24-tooth pinion and 93-tooth gear, with a diametral pitch of 32. Although undercutting is not technically a concern at 25 degrees contact angle, the tooth profile without addendum/dedendum adjustment is less than ideal, being narrower at the root than the maximum flank width (See Figure 12).

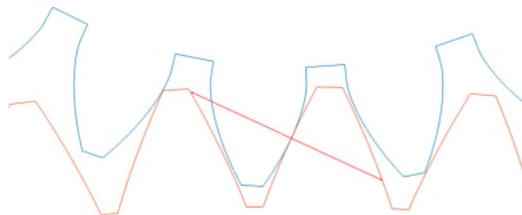


Figure 12: Small Gimbal Gear Teeth without Modification

For each contact angle, specific sliding, efficiency, and approach/recess action were calculated for various addendum and dedendum heights. In each case, the contact ratio was maintained at or slightly above the required minimum, in order to provide a fair comparison. These results are shown in

Table 1. The final design is highlighted in light blue and shown graphically in Figure 13.

Table 1: Specific Sliding, Efficiency, and Recess Action versus Addendum Modification

Ap	Ag	Max Specific Sliding	Mesh Efficiency	% Recess Action
1.0	1.0	1.00	95.2%	44%
1.1	0.9	0.87	96.0%	53%
1.2	0.8	0.73	96.6%	57%
1.3	0.7	0.60	97.1%	62%
1.4	0.7	0.54	97.3%	66%
1.5	0.6	0.48	97.4%	69%

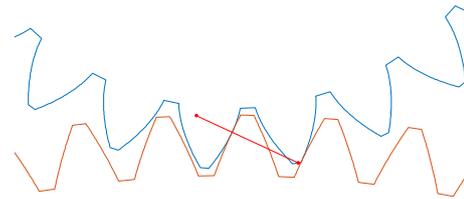


Figure 13: Final Gear Design

Center Distance Modification

Modification of the center distance can be particularly effective for eliminating undercutting or improving tooth form on gears with a low tooth count. Referring back to Figure 2, as the center distance is increased, the operating pressure angle increases, which effectively moves the mesh up the involute profile, allowing for a longer addendum and shorter dedendum without tip-to-root interference since the mating gear is pulled away by the increased center distance. Thus, the operating pressure angle can be increased to an arbitrary value, while still allowing conventional tooling with standard pressure angles to be used to manufacture the gear. Center distance modification can be combined with addendum/dedendum optimization and tooth width correction. To accomplish center distance modification, a modification factor is used, which is divided by the diametral pitch and added to the center distance. Therefore, a center distance modification factor of 1.0 increases the center distance by the same amount as adding an extra tooth to one of the gears.

Once the adjusted center distance is defined, the working pitch diameters and operating pressure angles can be defined, and the gear parameters can be calculated as per normal using these values.

For a pinion with a pressure angle of 25 degrees, 13 teeth, and a diametral pitch of 20, Figure 14 shows the tooth profile achieved without any modification, with long/short addendum, and additional addendum

modification enabled by center distance modification. Note that center distance modification enables a larger addendum on the driving gear, but due to the increase in operating pressure angle, provides only a modest efficiency gain (Table 2). The main advantage of center distance modification is the improved tooth geometry for bending strength and stiffness.

Table 2: Specific Sliding, Efficiency, and Recess Action versus Addendum and Center Distance Modification

Ap	Ag	Center Distance Modification	Max Specific Sliding	Mesh Efficiency	% Recess Action
1.0	1.0	0.0	1.81	91.2%	48%
1.2	0.8	0.0	1.16	94.6%	57%
1.4	0.8	0.2	1.06	95.0%	60%
1.4	0.6	0.0	0.74	96.4%	67%
1.6	0.6	0.2	0.69	96.5%	69%

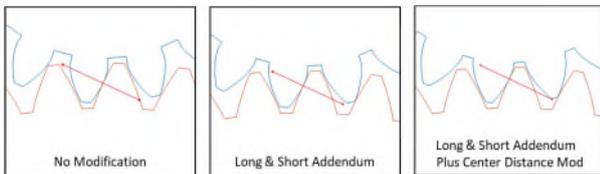


Figure 14: Tooth Profiles Achieved with Various Methods

Profile Modification

Per [3], profile modification should be performed in applications with relatively high tooth loads and requiring highly smooth or accurate operation. In coarser tolerance classes of gearing, the amount of material removed by profile modification may be smaller than the allowable profile deviations, tooth-to-tooth errors, etc. In other words, the tooth deflection amount should be compared to these tolerances before employing profile modification.

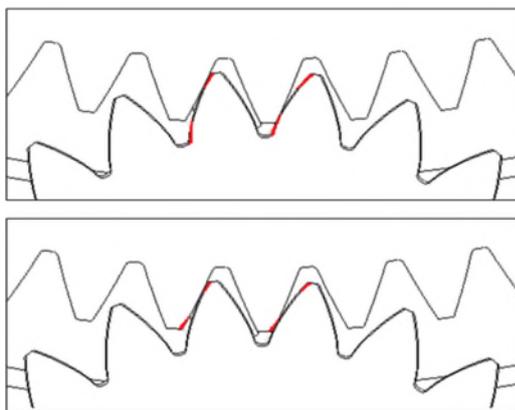


Figure 15: Tip and Root Relief Applied to 1 gear (Upper) & Tip Relief Applied to Both Gears (Lower) (Red shading shows where material is removed)

The basic approach of profile modification is to allow the tooth load to ramp up as a new tooth pair comes into

mesh, starting from zero, and reaching the full load once that tooth pair is the only one in mesh. If only one gear in the mesh will employ profile modification, then the profile should be modified at the tip and root. If both gears will employ profile modification, then profile modification can be employed only on the tips. See Figure 15. To determine the modified profile, first the tooth deflection is calculated as shown in Eq. (13) [3].

$$\delta_s \cong .0725 \cdot w_g \quad (13)$$

Where:

$$\delta_s = \text{approx. tooth deflection under load, } \mu\text{m}$$

$$w_g = \text{specific tooth loading, N/mm}$$

The profile modification should decrease to zero modification by the time only one tooth is in contact. Finally, profile modification should start again as the next tooth begins to mesh, reaching the full amount once the next tooth is the only one in contact.

To implement profile modification, roll charts are used. Roll charts can specify a variety of points, but the tolerance zone for the tooth profile can be defined with profile modification on a roll chart as shown in Figure 16.

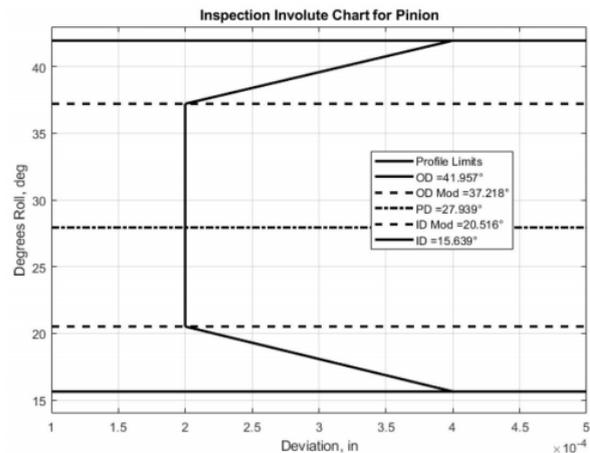


Figure 16: Example Roll Chart for Profile Modified Pinion (inch units)

Crowning

Crowning of teeth is a “longitudinal” profile modification which can be used when angular misalignment is expected to occur between gear axes. Whether caused by manufacturing tolerances or deflection under load, excessive angular misalignment will cause an uneven load distribution across the tooth and may even cause only a short segment at the end of the tooth to carry the full load. For gears with a long face width, torsional deflection can also contribute to an uneven load distribution [3]. To determine whether crowning is required, use the actual load distribution that

results from the expected angular misalignment when performing AGMA stress calculations. If the stresses are excessive, crowning can be used to eliminate this effect. [4] includes contact stress factors to include for crowned teeth. Figure 17 shows an example gear drawing specifying crowned teeth.

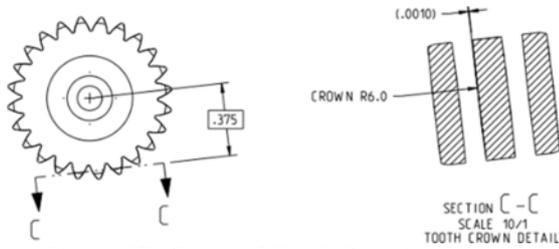


Figure 17: Crowned Tooth Drawing Example

Methodology

Although there are commercial gear design tools available, the involute equation is simple and it is not difficult to create MATLAB code or equivalent to facilitate gear design. This can be accomplished as follows:

To produce something similar to the code used for the examples shown here, start with basic input parameters: diametral pitch, tooth counts, nominal pressure angle, backlash amount and tolerance, center distance modification factor, center shift tolerance, tooth width modification factor, and addendum/dedendum values.

From these input parameters, calculate the base diameters, nominal pitch diameters, major and minor diameters, center distance, working pitch diameter, operating pressure angle, and contact ratio.

Using these parameters, calculate the profile for a single tooth flank, followed by the circular tooth widths, measurement over or under pins as well as pin diameter, (per [6] and [7]) and finally the profiles for each gear in the meshing region. Next, calculate the line of action followed by specific sliding versus gear angle, efficiency, and backlash. To get good plots, the “gears” must be aligned. This is accomplished by aligning the pitch point on each mating tooth to the Y axis. Then plot the gear mesh with the line of action and the specific sliding versus gear angle, and any other desired data.

These calculations (and plots) can then be repeated for each extreme case of minimum and maximum backlash and center shift.

The outputs provided to the user should be any desired plots as well as gear drawing parameters, contact ratio, efficiency, gear ratio, and min/max backlash. Such a script allows for rapid design iteration and optimization, as well as streamlining the calculations required to define a gear design on a drawing.

Similar scripts can also be created to perform the contact stress and bending fatigue calculations per [4] and [5].

MANUFACTURING CONSIDERATIONS

In order to generate producible gear designs, manufacturability must be considered. The authors have designed and produced a number of gear designs employing various degrees of modification, as described herein. The material used, wall thicknesses, and required tolerances will determine how the gears should be manufactured, as well as the achievable tolerances. When lead time is a driving factor, the use of standard cutters should be facilitated.

Material Selection

Material selection (from a manufacturability standpoint) is less critical for gears that will be ground. Hobbing is more flexible than shaping when it comes to material selection, due to better chip formation. However, if shaping is necessary on pre-hardened parts, the material selection is critical. Table 3 shows a ranking of materials for shaping. This ranking is based on a hardness range of HRC 45 to 50 with through-hardened material. Note that shaping is required for internal or external gears with any feature(s) that shroud the gear teeth without sufficient tool clearance for hobs or grinding wheels.

Table 3: Material Manufacturability for HRC 45 to 50

Material	Manufacturability Ranking	Comments
13-8 Mo	1	Minimal work hardening, better finishing, works well with fine pitch gears
Custom 455	2	Moderate work hardening, narrow tolerance for material removal on finish cuts
15-5 PH	3	Moderate work hardening, narrow tolerance for material removal on finish cuts
420 CRES	4	Significant work hardening, does not take finish cuts, not good for fine pitch gears

If harder gears are needed, gear grinding will be necessary. Carburized 9310 has been used for several recent space applications with success. Carburized gears require significantly longer lead times, since they have to undergo numerous additional steps: rough-cut (including gear teeth), copper plate on non-carburized surfaces, carburize, strip off copper plate, and finally finish grind. A few trial parts are usually required to ensure a full clean-up during final grind with a sufficient minimum and maximum case depth.

Wall Thickness

Thin-walled parts, especially internal gears, can be a challenge due to excessive deflection under load while

cutting. This is a particular concern when shaping, however it can be an issue when hobbing thin-walled external gears. For internal gears that can be ground, part deflection is less of a concern, but could cause issues at higher AGMA (2000) quality levels. It is not practical to provide a “rule of thumb”, as the cutting forces depend on material, hardness, diametral pitch, and so on, with deflection being dependant on wall thickness, part depth, diameter, etc. In general, the thicker the wall the better the finished part tolerances will be.

Use of Standard Gear Cutters

Standard gear cutters will usually have a whole depth of 2.25 divided by diametral pitch down to 18 diametral pitch, and 2.2 divided by diametral pitch plus some margin for 20 diametral pitch and finer. This correlates to a normalized whole depth of 2.2 or 2.25 depending on diametral pitch. For highly modified gears, standard gear cutters may not be useable due to insufficient whole depth or other factors. For these designs, custom tools and grinding are required for production, however, full-depth grinding has been performed successfully for prototype parts, eliminating the lead time of custom cutters.

Tolerances vs Manufacturing Method

For shaping and hobbing only, AGMA (2000) quality 9-10 is achievable when using workable materials with a sufficient minimum wall thickness. In applications requiring better than AGMA quality 10, gear grinding is required.

Manufacturing Method Guidelines

Quantities and pitch play a role in determining what manufacturing method is most cost effective. Low quantities lend themselves to grinding from solid to achieve AGMA (2000) quality 10 +. While this is achievable with the right controls from a hobbing perspective, grinding from solid is just as economical. Especially since the quality level is easier to achieve by grinding.

Where modified tooth forms are present it is also cost effective to grind from solid as a custom hob (cutter) does not need to be purchased. If pitches get too small, the grinding wheel may break down and, in this case, it is therefore more beneficial to hob to AGMA (2000) quality 10. Therefore, it is also process dependent.

CONCLUSIONS

This paper covered some basic tools and good references for gear design. Using these methods, issues such as undercutting, excessive gear vibration, and poorly distributed tooth loads can be reduced or eliminated. The gear tooth design can be optimized for efficiency and wear life.

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